ACOUSTICS

FOR

MUSICIANS

BY

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PREFACE

It is held by a good many people—and I am not concerned to contradict them—that the time spent by music-students in acquiring a knowledge of Acoustics is time wasted. The fact however remains that many students do so spend some of their time; and musical examinations, by asking questions on the subject, force them to continue doing so. And I suppose every one will agree that, if a subject *must* be studied, it is best to study it intelligently.

The experience of a good many years, both in teaching and examining, has convinced me that very few students ever succeed in grasping the underlying principles of Acoustics at all. They acquire the jargon, and store their minds with text-book facts; but they seldom grip the scientific basis on which the theory of sound is built. And in support of this contention I will advance two conclusions—it would be easy to give a dozen—to which I am driven by the answers to questions set in examination-papers. Firstly, it is obvious that the majority of students (I am speaking always of music-students) believe that wave-curves—so familiar to any one who has ever opened a book on Sound—are the actual pictorial representation of something which occurs in the air; and the true meaning of the essential word 'associated' has never dawned on them. Secondly, I have never yet been convinced by an answer to any question on equal temperament that the candidate really understood the bearing on the question of the twelfth root of two.

The truth is, of course, that the understanding of the principles of Acoustics, as distinct from the cramming of a number of facts, depends entirely on the grasp of a few elementary mathematical conceptions; and no book on the subject, so far as I can find, recognizes the fact that to the ordinary music-student mathematics, however elementary, is not familiar ground. So I have tried in this book to explain, in separate chapters, each fundamental mathematical idea at the point where the understanding of it becomes vital; and I have done my utmost to put such explanations into language which can be comprehended by any one whose knowledge of ordinary arithmetic goes as far as vulgar fractions.

Any student who can understand Chapters III, VI, X, XIII, XV, XVI, and XVII should find the rest of the book easy reading; but those to whom the above chapters are incomprehensible can never hope, in my belief, to understand the drift of the subject or the principles on which its laws are founded.

I have to thank Miss Townsend Warner, Mr. W. J. R. Calvert, and Mr. D. H. Nagel for their kindness in reading manuscript and proofs, and giving other help and advice.

P. C. B.

HARROW-ON-THE-HILL,
December, 1917.

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PRELIMINARY CHAPTER

When a man sets out to study the Art of Music he is, in all probability, under the impression that he is dealing with a homogeneous and isolated subject; but before long he will be forced to realize that his study, if it is to be in any sense far-reaching, must embrace a great number of subjects each of which is in itself merely a subsidiary and component part of a wide and comprehensive whole. It is easy and usual to speak of these subjects as if each one were, alone and by itself, a complete branch of the art and a fit field for the confined study of the specialist. But such a custom is both illogical and unwise: illogical because branches imply a trunk, and there is in music no central backbone from which the various subjects radiate, since music is itself but the sum of them; and unwise because the lack of 'musicianship', which is such a drag on the wheel of artistic progress in this country, is directly fostered by our habit of concentrating exclusively on one section of the art. We can imagine, for instance, an accidental gathering of four men, each of them eminent in the musical world, yet each skilled in such diverse directions that real conversation between them on music would be more difficult than on almost any other subject. One, let us say, is a virtuoso on the violoncello, another an expert voice-trainer, the third has an unrivalled knowledge of the school of Palestrina, whilst the fourth is in the front rank as a Wagnerian conductor. It is difficult to suggest a trunk on which to graft these four branches.

Fortunately, however, a considerable number of musicians realize the folly and danger of confining the attention within too small limits. To whatever special line we may be driven by our natural gifts or our daily bread, a point is soon reached at which our great need is study *off* that line if we are to fertilize our minds by enlarging our outlook. And so it happens that some musicians, looking round for subjects of secondary interest, are attracted to the study of Acoustics—the Science of Sound. Since, however, such musicians have in most cases not thought out for themselves the relation between this one aspect of music and the whole art, the succeeding paragraphs will attempt an explanation of that relationship, while incidentally unfolding the plan and order of arrangement of the subject which has been adopted in this book.

When a man receives a blow the occurrence may be described from any one of many points of view. We might ask a lawyer for a legal opinion on the assault, a boxing-expert for a technical description of the hit, a doctor for an appreciation of the damage, a moralist for a homily on self-control. But if we ask a psychologist for a detailed description of the incident as it strikes him *qua* psychologist, he will tell us that the blow itself was a *stimulus*, that the recipient then experienced an immediate *sensation*, followed at once (unless it was a knock-out blow) by the *perception* of the nature and cause of the sensation, and followed later by workings of the mind, which are called *concepts*, on the facts presented.

When we listen to music exactly the same process occurs. Something acts as a stimulus to our auditory nerve, producing the sensation of sound: there follows the perception of its nature, and we say to ourselves that it is a clarinet, or a barrel-organ, or C sharp; and the mind is immediately provided with material for concepts, and we pass into the realms of discrimination, memory, association, and so forth. If we look for the application of Acoustics to the above analysis we can see at once where it is involved. The whole question of the action of the stimulus is a matter for purely acoustical investigation;

the question of sensation is partly acoustical and partly physiological; the questions of perception and concepts are entirely outside Acoustics except in so far as the previous work of the sense-organ has presented us with sensations dissected and ready for the formation of conclusions.

In one important respect the action of the stimulus in the case of Sound differs from the blow given and received in the above example. For in the case of the blow the sensation is produced by direct impact; whereas in the case of Sound the actual cause of the sensation may be many miles away. But it is quite easy to eliminate this difference by imagining a case where two comrades stand together and one of them is hit by a bullet, the other merely hearing the report of the rifle. In the case of the man who is hit we can analyse the whole incident in certain water-tight compartments. There is (1) the producing-cause (the rifle), involving a study of mechanisms and explosives;

(2) the conveyance of the bullet from the rifle to the victim, involving a study of the trajectory of projectiles;

(3) the impact of the bullet, as a stimulus producing sensation;

(4) the subsequent effects, which bring the realization of disablement, the suggestion of self-preservation, and the vision of stretchers, bandages, and hospital.

The immediate interest of the wounded man would not, of course, embrace any such analysis, and his grasp of the situation would be summed up in the words 'I've been hit'. Similarly his companion's comment on the incident, as far as it affects the question of sound, would be the simple statement 'I heard the shot'. But the acoustical analysis of his experience falls into four precisely parallel divisions:

(1) The producing cause of the vibrations—i.e. the explosion;

(2) The conveyance of those vibrations through the air to the auditory nerve of the recipient;

(3) The impact of the vibrations, producing the sensation of sound;

(4) The subsequent effects, which enable the listener to deduce the nature of the weapon, the range and direction of fire, and any other facts which an expert may be able to determine from the characteristics of the sound heard.

Conforming to the above analysis, this book deals first of all with the production of Sound, and then proceeds to inquire into its three characteristics of Pitch, Intensity, and Quality. It then deals with the question of Temperament, which is the artificial adaptation of natural laws to practical use. The process of Transmission is then discussed, being purposely left to this later stage, since it is the part of Acoustics which involves by far the most difficult abstract conceptions. The last chapter of the book deals with the outlines of the physiology and anatomy of the Ear, and at this point the subject is abandoned, since the next step, which leads us to the brain, is the threshold of psychological and aesthetical considerations which lie outside the province of Acoustics.

The student is earnestly advised not to skip the occasional chapters dealing with mathematics (3, 6, 10, 13, 15, 16, 17). They contain nothing whatever which a reader of average intelligence and ordinary arithmetical knowledge should not readily understand; and, unless they are understood, the study of Acoustics becomes a mere committal to memory of facts, many or most of them misconceived. A grasp of the elementary mathematical basis on which the subject rests leads to an initiation into logical and inevitable processes, and an unforgettable apprehension of the principles upon which they work.

PART I. PRODUCTION

CHAPTER I

AXIOMS AND DEFINITIONS

NO one can discuss even the most elementary question of Acoustics without soon discovering the necessity of using technical terms. As in every other branch of learning, discovery and progress in our knowledge of Sound depend on our being able to express complex ideas in single words, in the certain assurance that those words convey to other people exactly the same meaning as they convey to us.

Some of the words thus used in Acoustics are specialized terms (such as 'rarefaction' or 'density') whose meaning will present little difficulty to any student. An idea has to be grasped and delimited, and then labelled with its proper name. But occasionally words are pressed into this specialized service (such as 'elasticity' or 'wave') which give great trouble to students because, though the words are used by everybody in ordinary conversation, they are used in a sense which is scientifically inaccurate.

In this chapter a short explanation will be given of the special use of the more common technical terms. But the student is warned that in most cases the explanation is incomplete, since a full understanding depends on a logical grasp of facts which, later on, are explained at length in sequence.

Sound. Sound is invariably caused by some kind of motion. It is not sufficient to say that sound is *accompanied* by motion, since motion is the preliminary condition which renders the existence of a sound possible, and any particular sound is, however momentarily, *subsequent* to the disturbance which causes it.

The motion of the body causing the sound is sometimes plainly visible, as in the case of the lower strings of the violoncello. In other cases, such as a tuning-fork or a tumbler struck gently with a knife, the motion is practically invisible to the naked eye.

Vibration. When a body moves continuously in one direction we say it is 'travelling'. But when it moves a certain distance in one direction and then moves backwards over the ground already covered, we say it is *oscillating* to and fro. If, for instance, we fix a strip of springy metal in a vice (as in fig. 1) so that it is exactly perpendicular, and then pull the top to the point *a* and let go, the metal strip will, in virtue of its springiness, oscillate between *a* and *a′* until it ultimately comes to rest in its original position.

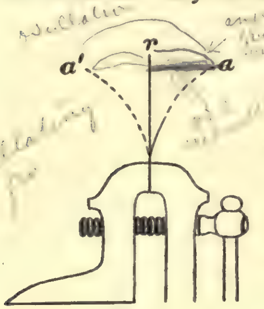


FIG. 1

The following facts in connexion with the above are essential and *must* be remembered:

(1) the movement forward from *a* to *a′* is an OSCILLATION[1]; so is the backwards movement from *a′* to *a*;

(2) the movement from *r* to *a*, *plus* the movement from *a* to *a′*, *plus* the movement from *a′* to *r*, is one VIBRATION;

(3) the distance from *a* to the perpendicular representing the state of rest is the AMPLITUDE of the vibration. This distance

[1] Unfortunately the meaning attached to such words as 'vibration' and 'oscillation' differs in different countries, and also in different writers in the same country. Students must not, in consequence, consider that the definitions given here are universally accepted, and in using the words themselves should state what meaning they adopt. It is very confusing that such a word as 'oscillation' should mean one thing in an English text-book and something definitely different in a French one. I have adopted the French meaning, as it seems to me simpler and more logical.

is found by drawing a line from *a* at right angles to the perpendicular, and since the point *a* is lower than the point *r* this line will meet the perpendicular at a point a little below *r*.

In the above experiment the distance *aa′* will gradually diminish, for two reasons. Firstly, the air-resistance, though very slight in each individual oscillation, will exercise a cumulative effect which by itself would in time produce a state of rest. Secondly, the elasticity (*vide infra*) of the metal tends to bring the strip back to its original position at *a*; but no metal is perfectly elastic and the oscillations consequently diminish in amplitude. If we imagine fig. 1 turned upside down, with a weight on a string replacing the metal tongue (thus eliminating elasticity), a state of rest would be induced by air-resistance alone. If we then eliminated this factor by swinging our pendulum in a perfect vacuum we could produce oscillations which would continue indefinitely if the whole apparatus were free from friction and the effects of wear and tear.

Any body which is vibrating is called a *vibrating body*, and the *vibration-number* of a body is the number of times it performs a complete vibration in a second. This latter is also sometimes called the *frequency* of the vibration.

Periodic. Movements are called *periodic* when they are repeated so as to occupy exactly equal periods of time.

If a conductor beats time for 60 bars of music which is in $\frac{2}{4}$ time and is marked ♩ = 120, his beats will be periodic if they are mathematically exact. His baton will describe 60 complete vibrations in the minute, and its vibration-number will be 1 per second. If, however, the passage included an *accelerando* and *ritardando*, the beating will not be periodic, even though by careful balancing the final note is reached exactly 'on time'.

Medium. The vibrations of a vibrating body do not reach a listener direct, but are communicated to a *medium* which in turn conveys them to the listener. This medium is almost invariably the air, and so far as musicians are concerned with Acoustics no medium other than air is of any importance. But almost anything *may* be the medium—wood, water, lead, steel, gold and silver, &c.—provided that elasticity is a property of whatever is chosen. And in some respects air is actually inferior to certain other media. If a tree-trunk of some length is lying on the ground it is easy, by applying the ear to one end, to hear quite distinctly the sound caused by some one at the other end gently scratching the wood with a knife—a sound which is quite inaudible to any one standing up (i.e. relying on the air for conveyance) at half the distance. The experiments made with bells, &c., under water may be mentioned because, though unimportant to musicians, the results have led to the invention of apparatus for warning ships of the presence of icebergs and for disclosing to submarines the neighbourhood of other craft.

Velocity. The rate at which vibrations will travel from the original vibrating body to the ear of the listener is called the *velocity* of sound in whatever medium is chosen.

In air the velocity of sound, when the temperature is at freezing-point (0° Centigrade or 32° Fahrenheit), is taken as 1,090 feet per second.

The pace of sound increases as the temperature rises, but does not increase at a uniform speed. But as the velocity at 80° Fahrenheit (an increase of 48° on freezing-point) is 1,140 feet (an increase of 50 feet per second on 1,090), it may be taken as roughly true that 1 foot per second is to be added to the velocity of 1,090 for every degree of heat above freezing-point Fahrenheit.

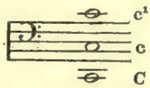
In water the velocity is nearly 5,000 feet per second; in wood (along the grain) from 10,000 to 15,000; in metals, such as iron and steel, it reaches over 16,000.

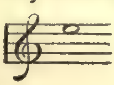
Velocity is not altered by atmospheric pressure: so long as the temperature remains the same the state of the barometer is immaterial. It is also independent of the pitch and loudness of the sound: the louder the sound the *farther*, but not the *faster*, it will travel.

Pitch, Intensity, and Quality are the three determining characteristics of every musical sound. That is to say, when these three qualities have been ascertained the character of any sound can be finally fixed.

High and Low, Sharpness and Flatness. There is no reason in the world why, if we strike two notes on the piano, we should say that the right-hand note is *higher* than the other. It is no nearer the ceiling, and though, as the student will learn later, its vibration-number happens to be greater, on the other hand its pipe-length and string-length are less. Yet it seems to be universal to use the words 'high' and 'low' in this sense. The Greeks used ὀξύς and βαρύς, and the Romans *acer* and *gravis*; and it is undoubtedly the same instinct which leads us to say, when a note is slightly higher or lower than we wish, that it is 'sharp' or 'flat'. The student should be quite clear in his mind, however, that such terms, though used always with one accepted meaning, are picturesque rather than rational.

Nomenclature of Notes. Everybody should be familiar with the method of naming notes so as to convey their 'octave' as well as their name. Low C (i.e. bottom C on the pedal-board of an organ at 8 ft. pitch) is called C, the octave below C_1 or CC, and the octave below that (32 ft. C) C_2 or CCC. In the other direction tenor C is *c*, middle C is *c′* or *cc*, and so on. Every note in any octave is named in the same way as the C *below* it.



Thus  is *e″*;  is B_1 or BB.

Condensation and Rarefaction. If we have a tumbler full of water, and then take half the water away, the result is that the tumbler is half-full; and the same result would follow if the tumbler were air-tight. That is to say, the lower half would be full of water, the upper half becoming a vacuum, since the water would not expand. In the case of air, however, we should get a different result. For if we withdraw half the air from an air-tight chamber, the remaining half expands and fills twice its normal space, and there are no empty 'pockets' in the chamber. When air expands in this manner into *more* space than it requires in its normal state we say that it is 'rarefied'; when it is compressed into *less* space than it normally requires we say it is 'condensed'. The comparative ease with which air condenses and rarefies is of the utmost importance at a later stage of the subject.

Water-wave. This word immediately suggests, to the great majority of people, something moving along in a curly sinuous manner. The essential point of all water-waves, however—and the realization of this is of cardinal importance when we come to 'associated' waves—is that, though the movements that take place *seem* to be in the direction of the wave, they are all really at right angles to this direction.[1] If, for example, we place a walking-stick underneath the table-cloth, and move it along the length of the table, it is obvious that a wave passes down the cloth; but all that the actual particles of cloth have done has been to rise and fall perpendicularly on the table, at right angles to the direction of the wave which results from their movement. It is quite possible, indeed, that a wave-system may be in operation in which the actual *material* of the waves is momentarily moving in a direction opposite to that of the wave-system. The student should from the outset make himself familiar with this idea that the directions of wave-motion and material-motion are quite independent.

[1] When the student makes acquaintance with Weber's Law (p. 109) he will slightly modify this view, but at the moment this qualification is not of importance.

Volume, Mass, and Density. The *volume* of a substance means the cubic space which the substance occupies.

The *mass* means the quantity of matter, and is proportional to the weight.

The *density* of a substance is the relationship of the mass to the volume. This is usually expressed by the formula

$$D = \frac{M}{V}.$$

This formula will become clear to any one who will apply it to a simple concrete example.

If we find that 10 cubic inches of a certain substance weighs x pounds, then by taking a pound as our unit of weight and a cubic inch as our unit of volume we can find the density of that substance:

$$D = \frac{M}{V} = \frac{x}{10}.$$

If we halve the volume we also halve the weight, but do not affect the density, for still

$$D = \frac{M}{V} = \frac{\frac{1}{2}x}{5} = \frac{x}{10}.$$

If we *compress* the substance into *half* its normal space then we *double* the density, for it still weighs x pounds, and so

$$D = \frac{M}{V} = \frac{x}{5}.$$

If we could so treat the substance that it lost *half* its weight without decreasing in size, then we *halve* the density, for

$$D = \frac{M}{V} = \frac{\frac{1}{2}x}{10} = \frac{x}{20}.$$

Thus the normal density of any substance is found by fixing on some definite unit of weight and unit of volume (or size), and comparing the two in the form of the fraction

$$\frac{\text{units of weight}}{\text{units of volume}}.$$

Any increase of weight while the volume remains stationary obviously increases the value of this fraction—i.e. increases the density of the substance; any increase of volume while the weight remains stationary obviously decreases the value.

Elasticity. Almost every one, except those with scientific knowledge, connects the idea of *elasticity* with the ease with which a substance allows itself to be stretched. The true meaning of the word is quite different. A body possesses elasticity in proportion as it demands force to change its original form, and insists on recovering its original form after it has been disturbed. It is not easy at first to realize the great elasticity of glass, which will almost instantly fly back to its original position when it has been bent; whereas many substances we should name at once as obviously elastic possess in reality but little elasticity, as they are easy to bend and sluggish in recovering their original forms.

Tension. If we hang a weight on to a string the tension of the string is at all points exactly equal to the weight; and the same is true if we apply a stretching-force to the string of a musical instrument. Hooke's Law (*ut tensio sic vis*—that the force is as the tension) established the fact that the increase in length is proportional to the force applied at the ends. Consequently if an extra force of x pounds is required to make a violin-string stretch one more inch, then a force of $2x$ pounds will stretch it two inches in all; and the tension at every point of the string is equal to the total force applied.

N.B.—So many of the technical terms of Acoustics have synonyms, and the student is so apt to be confused by a term with which he is not familiar, that it has been thought well, when two terms mean the same thing, to use them indiscriminately in this book. Thus the word 'frequency' is used as often as 'vibration-number'; and 'pure sound' and 'simple tone' are another pair; also 'timbre' and 'quality', 'wave' and 'curve', 'intensity' and 'loudness', and, in the latter half of the book, 'fundamental note' and 'prime tone'.

CHAPTER II

THE PRODUCTION OF SOUND

ALL language aims at establishing a general understanding between people, and in consequence every one acquiesces in expressions which are not scientifically accurate, so long as they serve the purpose of facilitating the exchange of ideas. We have in this way grown into the habit of using the word 'sound' in a sense quite divorced from its scientific meaning, allowing our minds to jump proleptically from a means to an end. If we say, for example, that 'a bugle is sounding', we make a remark which is unmistakably intelligible to every one; but it is nevertheless, from a scientific standpoint, entirely inaccurate. For the bugle merely *vibrates*, and the vibrations which it communicates to the air are not translated into sound until they come into contact with some auditory apparatus. An ear of wheat can produce flour which man may turn into bread, and the poet, holding the ear in his hand, might apostrophize it as bread; but it obviously has not yet become bread, and possibly it never will. Similarly the bugle can produce vibrations in the air which a listener's ears may turn into sound, though possibly this transformation will never take place: as would be the case were a stone-deaf bugler to blow his instrument out of the hearing of the rest of the world. Again, we should say in conversation that if you shake a bell it 'sounds'; but a simple experiment will show that this is not necessarily the case. If you place a bell under a glass jar, and then set it in motion, it is true that in your head the vibrations coming *from* the bell *through* the air are turned into sound; but if you exhaust the air in the jar, until the bell is swinging in a vacuum, the result is silence, because the vibrations have no means of reaching your ear. Yet the bell is performing precisely the same part as before.

The conception of sound embodied in the above paragraph is so essential to a right understanding of all acoustical phenomena that it is worth while to dwell on its bearings a little longer.

Supposing that you see a big gun fired some miles out at sea. First a puff of smoke will appear, and some seconds later you will hear the report of the explosion. At the moment of hearing this sound, ask yourself *where* it is. It cannot be at the mouth of the gun, for if you could telephone to the gunner he would say that in the neighbourhood of the gun there had been peace and quiet for some seconds. It clearly cannot be at some chance point between the gun and you. Consequently the sound must be in your head. And though other similar sounds will be in the heads of all the other listeners yet, if we assume for the moment that the gun was fired by electrical contact on a desert island with yourself as the only soul within range of hearing, then we can say that the *only* sound for which the gun was responsible was that one particular sound in your head.

A commoner and perhaps more striking illustration is the following. If you go alone to the organ in an empty church, pull out all the stops, fix down all the notes, and turn on the wind, the result will be a noise best left to the imagination. Then go and stand outside the church, leaving it empty as you found it. You will still hear the organ, but in the church itself there is *perfect silence.* The organ-pipes are vibrating and are doing nothing else; the air is passing on those vibrations to the nearest ear, and is doing nothing else; and the first *sound* in connexion with the whole affair arises at the moment when, and the place where, the vibrations come in contact with a living apparatus designed to receive and translate them.

When a body is vibrating it is not always, or even generally, possible to see the vibrations with the naked eye. In some cases—stretched strings, for example—the movement is obvious; in others, such as tuning-forks, it is invisible in small-sized forks though noticeable in large ones. But even in the case of small forks the existence of rapid movement is quickly established if, after striking the instrument on something solid, we apply it to our tongue or lips. When we accidentally strike a tumbler during a meal we show, by placing a finger on it, that we know the sound will cease if the glass is reduced to a state of rest. If, again, we draw a sound from a finger-bowl half-full of water, by rubbing a moistened finger gently round the rim, the vibration of the glass is established by the visible excitement of the surface of the water. It is, however, impossible by the unaided examination of the eye to form any reliable conclusions as to the nature of vibrations, and various methods have been invented for bringing them more under observation. Two of these methods will be described now.

I. *The graphic method.* A fine point is attached to one prong of a tuning-fork, and a strip of glass is prepared by blackening it in a flame. The fork is set in motion and the attached point applied to the strip of glass, which is then moved along at a uniform pace. The point traces a curve on the glass by scratching away the black as the prong moves to and fro. [In the illustration the curve is shown, for the sake of clearness, as black on white instead of white on black.]

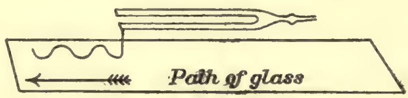


FIG. 2

II. *Koenig's flames.* Sometimes called the manometric flame. A is an ordinary gas-jet supplied with gas through the tube at B. This gas enters the oval enclosure and can only escape at A, since it is prevented from reaching C (the only other exit) by the membranous partition at DE. Now a note sung near the apparatus will make the membrane vibrate in sympathy with it; and each forward or backward movement of the membrane will make

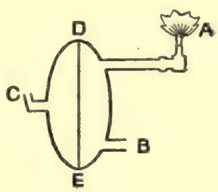


FIG. 3

the flame rise or fall as it presses the gas towards the flame or pulls it in the opposite direction. This up-and-down motion of the flame being too rapid to see, it is reflected in a revolving mirror, which makes it easy to obtain certain conclusions. The most important of these is the fact that, whereas a noise will produce in the mirror a series of jagged flames of all sizes and shapes, a musical note produces a series characterized by symmetry and smoothness of outline.

It is necessary to establish on a scientific basis the difference between a musical sound and a noise, and a line of demarcation has been drawn by scientists in accordance with the conclusions arrived at from physical experiments such as the two just described. When the vibrations of a body recur at exactly regular intervals they are said to be periodic, and such periodic vibration will produce a musical sound; whereas irregular unperiodic vibration will invariably result in noise. As will be found later on, such periodic vibration results in definiteness of pitch, and so a musical sound is considered, scientifically, as one that has an ascertainable pitch, and the question of pleasantness of quality is ignored. The rustling of leaves, for instance, might be described by a poet as musical, but to the scientist it is to be classed with noises.

There are, nevertheless, some doubtful cases. A cork pulled out of a bottle, or a pencil pulled sharply out of a case, frequently give notes of clear pitch. On the other hand the cymbals and triangle, both used for musical purposes, have no pitch at all; the triangle, indeed, possessing the baffling property of apparently sounding in tune with whatever note is being played with it. It will be well, however, to accept the scientific division, since it is meant to establish a working classification and not to enunciate an inexorable law.

A body vibrating periodically at a continuous and uniform rate will always (as experiment proves) produce sounds of the same pitch; and any other body made to vibrate at exactly the same rate will produce a sound in unison with the first. Hence it is established that every note has an exact number of vibrations per second required to produce it, and, when the right number for a given note has been ascertained, any body made to vibrate at that rate will inevitably produce the note in question. The number of vibrations required to produce a note is called the note's *vibration-number* or *frequency*, and we may describe the pitch of a note *absolutely*, by giving its vibration-number, or *relatively* by comparing it (as fifth, octave, &c.) with another note.

From the earliest ages man has tried to increase the number of ways at his disposal for producing periodic vibrations, and to enlarge the number of sounds obtainable by any one method. At present there are six common ways of generating sounds for musical purposes:

(1) By the vibrations of columns of air (e. g. the penny-whistle, which is the type of all flue-pipes).

(2) By the vibrations of stretched strings.

(3) By the vibrations of reeds.

(4) By the vibrations of elastic membranes acted on by air (e. g. the human voice).

(5) By the vibrations of elastic membranes acted on by solids (e. g. the drum).

(6) By the vibrations of elastic solid bodies (e. g. bells).

PART II. PITCH

CHAPTER III

ON PENDULUM-MOTION

If we attach a weight to a cord which is fixed at the other end, and then set the weight in motion by making it swing backwards and forwards in a plane (i. e. without circular motion), we have an example of a simple pendulum in action. The weight is called the 'bob', and the path of the bob through the air, which is clearly a portion of the circumference of the circle of which the fixed point is the centre, is called the arc of vibration.

The actual movement of a pendulum is so simple that the reader will have no difficulty in imagining one in motion. A garden swing, if we assume that the two ropes keep exactly parallel, and so traverse precisely similar paths, will furnish an example. We take hold of the bob (i. e. the seat and the person sitting on it), force it away from the perpendicular position it assumes when at rest, and then let go our hold. If no further force is applied, either by ourselves or the passenger, the pendulum will oscillate backwards and forwards, describing an arc which grows less with each oscillation until ultimately the original position of rest is gained.

The one essential feature of these oscillations, which is called the Law of Pendulum-motion, is that so long as the arc of vibration is small the oscillations are isochronous; that is, they occupy exactly the same time. Between the full swing (which is just short of a semicircle) and the smallest (which is the minute fraction of an inch immediately preceding absolute stoppage) there is a discrepancy in time of something like 20 per cent.; but so long as the arc remains small—and for musical purposes (cf. the tuning-fork) only small arcs are necessary—oscillations are isochronous and are not affected in time by the diminishing amplitude of the arc. We can alter the pace of a pendulum by lengthening or shortening the cord, but when once the length is fixed any given pendulum will beat time with mathematical exactness, provided that it is started on its swing with a fairly small arc.

Ever since the law of isochronous motion was discovered by Galileo experiments have been made with a view to gaining further insight into the nature of pendulum-motion. One of these experiments, of the very greatest importance in Acoustics, is the following.

A pendulum is provided with a bob which is a hollow sphere filled with a fine powder; and as the pendulum swings the powder escapes (like the sand in the top half of an hour-glass) through a hole in the bob. If a level board, large enough to cover the swing of the bob, is placed underneath the pendulum, it will obviously receive the powder in a straight line. But if the board is moved along at a uniform pace in a direction at right angles to the movement of the pendulum, then the sand will be found to have traced a curve on the board.

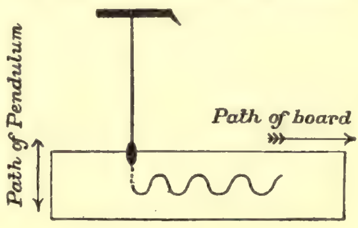


FIG. 4

The curve thus formed (fig. 4) represents to the eye the movement in a simple pendular vibration, and it is found, on examination, to be of the same type as the curve traced on smoked glass by a tuning-fork (fig. 2, p. 23). It is not necessary at this point to dwell on the deductions to be drawn from this coincidence, beyond saying that they are of such importance as to make it imperative for the student to understand the nature of pendular vibration. And the cardinal feature of the whole process is the fact, as stated above, that though the curve described by the bob grows ever less and less, yet the time occupied by the bob over each curve, great or small, is exactly the same.

Many musicians have proved for themselves the truth of this law by constructing simple and inexpensive metronomes. A small weight attached to a tape and hung on a nail in the wall will, when set in motion, from its first swing (which must not be of too large a sweep) to its later scarcely perceptible movements, keep absolutely strict time. The fixed laws for altering the pace of such a metronome, by changing the length of tape, are quite simple, but as they involve inverse variation and square root they will be explained later on (Chap. VI, p. 39).

It may be desirable to go a little beyond the above superficial account of pendulum-motion, for the benefit of those readers who wish to have a more scientific conception of the process.

When the bob is held away from the position of rest and set free it falls, from rest, with increasing speed towards its position of rest.

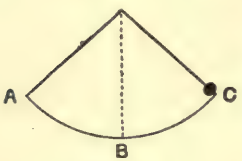


FIG. 5

E.g. in fig. 5 if we hold the bob at C and then set it free, it falls with increasing speed towards B. It then swings uphill with diminishing speed to A, where it comes to momentary rest; and proceeds to do the same journey in an opposite direction. There are two forces to be considered, which affect the pendulum:

(1) *Gravity*, which causes the movement from C to B; the momentum thus acquired would (in the absence of other forces) work against gravity and take the bob to a point A exactly as far from B as C is.

(2) *Friction*, of various kinds (including air-resistance), which is the factor that causes the gradual shortening of the arc and the ultimate state of rest of the bob.

Hence the distance of A from B is a little less than that of C; and the bob will swing back to a point near C, but *slightly* nearer to B than A is. If a pendulum could be constructed free from any kind of friction, and were set swinging in a vacuum, it would go on for ever.

The layman is apt to look on 'energy' as necessarily implying motion; but it is really of two kinds:

(1) *Potential energy*: that is, power to do work by reason of position—e. g. a book lying on a table.

(2) *Kinetic energy*: that is, power to do work by reason of motion—e. g. a book falling on to the floor.

In pendulum-motion the movement from C to B (fig. 5) involves a gradual change from potential to kinetic energy (which is complete at B); from B to A a reverse gradual change from kinetic to potential (which is complete at A).

Although the isochronism of pendulum-motion is the essential point affecting Acoustics, and the understanding of it the whole object of this chapter, it should be stated, in the interests of accuracy, that the cycloidal pendulum is the only one which is perfectly isochronous. The error, however, in the ordinary pendulum is not great where the difference in arc-lengths is not considerable, and the musical student need not modify his conception unless he intends to penetrate fairly deeply into the mathematical and physical basis of the subject.

CHAPTER IV

ABSOLUTE PITCH

WE have now three facts at our disposal on which we can build our ideas of pitch:

(1) A steady rate of vibration results in a note of steady pitch.

(2) Pendulum-motion, within the limits necessary for acoustical purposes, is isochronous ~~or periodic~~.

(3) The vibrations of a tuning-fork are of a pendular nature.

FIG. 6

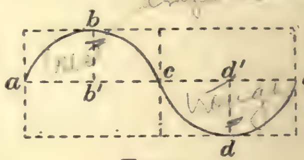


FIG. 7

A glance back at fig. 2 (p. 23) will recall the fact that a vibrating tuning-fork can be made to trace its own curve, which is like that in fig. 6. The portion of the curve enclosed in the oblong of dotted lines is that traced by the needle affixed to the prong during one complete vibration. The curve begins at the point the needle would occupy when at rest, moves upwards to the full extent of its swing, downwards to the full extent in the opposite direction, and upwards again to its point of rest.

Examining such a section in detail (fig. 7), we can establish various facts about the vibration of the prong which traced it.

(1) *abcde* is the curve representing one complete vibration.

(2) *abc*, *cde* are curves exactly similar, but on different sides of *ae*.

(3) *ace* is the *length* of the vibration-curve.[1]

(4) *abc* is called the *crest*, *cde* the *trough* of the curve.

(5) *bb'* or *dd'* (which are equal) give the amplitude of the crest and trough.

Now a very little thought applied to the experiment in fig. 2 (p. 23) will make it clear that, as the vibrating fork approaches nearer and nearer to its point of rest, so will the curve traced on the glass approach nearer and nearer to a straight line. The same fork which, in the full vibration following a blow, will trace the curve in fig. 7, will some seconds later, when the initial impetus has weakened, produce the curve of fig. 8. And the cardinal fact which a comparison of figs. 7 and 8 reveals is that, though the *amplitude* of the curve has changed, the *length* remains constant. This is the fact which we learn from the application of the law of pendulum-motion to the vibrations of a tuning-fork: that if the smoked glass be drawn across the path of the attached needle at a uniform pace, then no matter how large or small the amplitude of the curve traced may be, it will be found by measurement that the length of each successive wave remains constant.

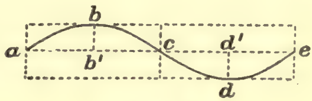


FIG. 8

The same treatment may be applied to the experiment with a strip or tongue of metal fixed in a vice (fig. 1, p. 14). In fig. 9 the perpendicular line represents the tongue of metal, fixed in a vice at V. It is forced from its point of rest at *r* to the point *a*, set free, and swings

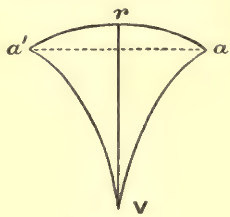


FIG. 9

[1] It may well be noticed here that there is a definite relationship between this length and the rate of vibration. If, for example, the tuning-fork whose prong traced the curve of fig. 6 had vibrated twice as fast, then there would have been twice as many vibration-curves on any given length of the smoked glass: i. e. *each vibration-curve would have been half as long.*

across through *r* to *a′*. The amplitude of vibration is half the distance *aa′*, and it is clear that, as the tongue gets more and more tired and nearer to its state of rest, so the amplitude gets smaller and smaller. But the vibrations remain periodic, each taking the same time independently of the changes of amplitude.

If we increase the rate of vibration of a vibrating body we find that the pitch rises; if we decrease the rate the pitch falls. And it is an established fact that pitch depends on nothing else than this rate of vibration. Hence, of two notes, the one that is higher in pitch has inevitably the higher vibration-number; and the vibration-number of a note means always the number of vibrations per second that will produce the note. The fraction of a second occupied by one vibration is called its *period*; and if we know *either* the period of one vibration *or* the number of vibrations in a second, we can find the unknown from the known by means of the simple formulae

$$T = \frac{1}{N} \text{ or } N = \frac{1}{T},$$

where T is the time of a period and N the number of vibrations per second.

It is possible to test, by mechanical means, the limitations of audibility in a person; and such tests reveal curious divergences between individuals in the power both of recognizing pitch and of hearing extreme sounds at all. Incidentally, it is not so widely recognized as it should be that these two faculties are quite distinct, bearing to each other a relationship analogous to that between the power of sight and the power of distinguishing colour. If we watch the departure of a ship with red funnels, the colour will cease to be distinguishable long before the funnels become invisible; and if we listen to a chromatic scale which goes on up or down without stopping we soon reach a point where pitch is unrecognizable, though for some time we continue to hear squeaks or grunts.

The Galton whistle is an instrument invented for testing in the region of high sounds. By shortening the pipe (and in consequence increasing the rate of vibration) it can be made to produce a sound which rises continuously higher. At a certain point a listener will fail to hear anything at all, and will claim that the instrument is silent. But the whistle is really producing its vibrations just as before, only so rapidly as to be useless to the ear of the particular listener; whereas another individual might still find the sound within range.

There is a curious difference in the feeling we experience when the silence-point is reached at the low end of the scale. At the high end, when a number of listeners all agree that there is silence, there is nothing further to be registered. At the low end, however, we reach our deepest note, and then the sound disintegrates, not into silence, but into a system of throbs. Just as in a cinematograph, when the pictures are presented at too slow a pace, the eye fails to produce the illusion of continuity, so the ear, failing to connect the too-slow vibrations into a continuous sound, registers them only as a series of disjunct sensations.

It is impossible to give any very definite limits to the power of the human ear to recognize either pitch or sound, since, as has been stated, this power varies considerably in individual cases. But as a general rule it may be said that no note whose vibration-number is lower than 20 or higher than 38,000 is audible, and that pitch cannot be recognized unless the vibration-number falls between 30 and 4,000.

CHAPTER V

THE MEASUREMENT OF PITCH

As soon as the elementary facts of Acoustics were discovered—we might almost say as soon as they were suspected—men began to make use of them, as is the custom of human beings, in two ways. Men with a practical turn of mind set to work synthetically to construct instruments which would produce vibrations of the periodic kind, and in course of time found out the six methods tabulated on p. 25. Those men, however, whose minds were of an inquiring and scientific bent wished rather to analyse and group the facts, and thereby to penetrate farther into the unknown. Consequently their first object was to devise various ways of bringing under observation the vibrations which they knew to be too rapid and too minute for the unassisted senses to grasp. The methods thus invented for the measurement of pitch, though at first necessarily rather crude, have now reached a state of great ingenuity and perfection, and may be grouped under four headings:

(1) mechanical,
(2) optical,
(3) electrical,
(4) photographic.
5 graphic

Mechanical. The two chief methods are (*a*) the toothed wheel and (*b*) the siren.

(**a**) Savart's toothed (or ratchet) wheel consists of a circular disk of metal with equidistant teeth cut into the circumference. The disk is fixed into an apparatus which will keep it revolving at any pace desired, and this pace can be kept perfectly uniform and can also be exactly ascertained. A thin strip of metal (or even card-board) is fixed so as to touch the teeth of the wheel as it revolves; and the elasticity of the strip must be such that its tendency to recover its position will result in each separate tooth striking it in turn. Each up-and-down motion of the strip will constitute one vibration, which will be communicated to the air; and by changing the speed of revolution a note of the desired pitch can be secured. The vibration-number of this note is then found by multiplying the number of teeth in the wheel by the number of revolutions per second. Thus, if there are 100 teeth in the wheel and it has revolved exactly twice in a second, it is clear that the strip has been hit 200 times and the resulting note has a frequency of 200.

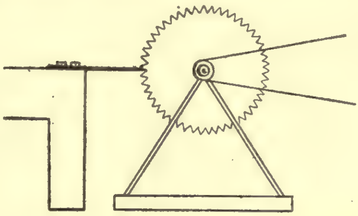


FIG. 10

For the sake of greater accuracy this calculation is generally made by taking the number of revolutions *per minute*, and dividing the result by 60. Thus if $T =$ the number of teeth, and $R =$ the number of revolutions per minute, we can find the vibration-number (V) of a note of any pitch from the formula:

$$V = \frac{T \times R}{60}.$$

(*b*) The siren is an instrument of exactly the same nature as the toothed wheel, with the one difference that the vibrations are caused directly in the air instead of being communicated to the air by the motion of an elastic body.

A disk is taken as before, but instead of cutting teeth on its circumference we pierce a series of little circular holes in it, all the holes being equal in size, equidistant from each other, and all at the same distance from the centre. The disk is placed in the same apparatus and made to revolve. We then blow through a tube held steadily in position so that the column of air blown through it would

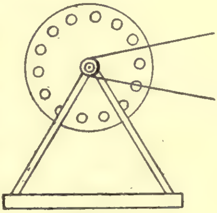


FIG. 11

pass through one of the holes when the disk is at rest. As soon as the revolution begins this column of air is cut up into 'puffs', as it is alternately allowed to pass through a hole and then is cut off until the next hole appears opposite to the end of the blow-pipe. Each puff creates a vibration in the air, and the vibration-number of the resulting note is found, as in the case of Savart's wheel, by multiplying the number of holes and the number of revolutions.

It is not proposed to describe in detail any other methods of measuring pitch, since they concern mathematicians and men of science rather than musicians. They owe their existence to the need for reducing the margin of inaccuracy, which is necessarily rather wide in the more elementary methods, as near as possible to zero. But a few of these inventions will now be mentioned, as the student should have a general idea of the scope of scientific investigation.

Mechanical methods include, besides the two already described,

(1) the *Vibroscope*—which is based on the graphic method (see fig. 2, p. 23);

(2) the *Monochord* or *Sonometer*, an instrument with a single string, one end of which is fixed, the other end being attached to a weight. This is not primarily an instrument for determining pitch, and results obtained from it involve the use of π; consequently it would serve no purpose to discuss it further;

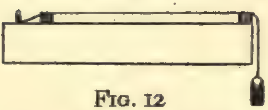


FIG. 12

(3) measurement by means of the *phonograph* and kindred instruments.

Optical methods:

(1) *manometric flames*, already mentioned on p. 23;

(2) *M. Lissajous'* method of making visible the vibrations of a tuning-fork;

(3) the *Cycloscope*, an elaborate combination of microscope, revolving black drum marked with equidistant white lines, and a tuning-fork which, entering the field of vision, creates waves which the expert can control and observe.

One other invention may be named—the *Tonometer*—which by means of either reeds or tuning-forks provides data on which mathematicians can base their calculations. The exact nature of the apparatus, which involves a familiarity with the problem of 'beats of the third order', can be discovered in any work which treats Acoustics from the purely scientific side.

The pitch of any given sound is determined and expressed without any ambiguity by means of its vibration-number; but the pitch of any given note in the musical scale is a matter of pure convention. It has always been evident that great conveniences would result from the existence of one recognized *Standard of Pitch*; but it is only in recent times that serious efforts have been made to fix such a standard.

Theoretical writers have for long favoured a pitch which assumed an imaginary note C whose vibration-number was 1. This gave a simple system of units. One vibration took one second: one wave-length was equal to the number of feet in the velocity of sound for one second, &c. Then again the first octave of this note would have the vibration-number 2, the second octave 2^2, the third octave 2^3, and so on, until we reached middle C (2^8) 256 and treble C (2^9) 512. This standard is still frequently used by mathematicians, and was formerly called *Philosophic Pitch*.

It is known that conventional musical pitch has risen consistently from early times. Handel's tuning-fork (to go back no farther) gave treble C 510, but the old 'concert-pitch' in England gave 528 for the same note,[1] and the pitch recognized by the Philharmonic and Crystal Palace orchestras, until the end of last century, was as high as treble C 538, and Covent Garden, in 1878, adopted 540.

In France a Government Commission, in 1858, fixed treble C = 517, and this is called French pitch, or Diapason normal;

[1] Fixed by the Society of Arts in 1869.

and in England, in 1896, almost the same pitch was adopted—522—which is now known as New Philharmonic. This standard is gradually becoming universal, and the only obstacle to it of any account is the expense involved in replacing wind-instruments and retuning organs.

PART III. INTENSITY

CHAPTER VI

ON MATHEMATICAL VARIATION

WHEN we say that a thing varies in amount we mean, as every one knows, that the quantity of it is different at different times. Sometimes such variation seems to be quite arbitrary, and we can discover no cause for it, and no method of estimating what the quantity will be at any given time. But in other cases we can see that the increase and decrease which constitute the variation are due to certain laws, and that if we understand the laws we can predict the variation with certainty. In such cases the increase and decrease are found to be indissolubly connected with the increase and decrease of something else; and it is one of the tasks of mathematicians to discover the relation between the two rates of variation. A student's progress, for instance, depends on the amount of work he does—i. e. the variation in progress is indissolubly connected with the variation in work, each (as mathematicians would say) being a *function* of the other. And though such a case is one where everybody will instantly see the connexion, it is also one where it is peculiarly difficult to discover a definite relation. For though eight hours' regular work a day may produce twice (or more than twice) the progress resulting from four hours' work, it is clear that twenty-four hours, indulged in regularly, will not produce three times the progress resulting from eight.

There are four kinds of mathematical variation which occur in elementary acoustics, and a student should be able to grasp their meaning with very little trouble. The first two kinds are merely matters of obvious common sense; but the two last, though almost equally simple in meaning, can only be expressed in what, to most people, is mathematical jargon. Hence these two latter forms are seldom really apprehended by those unfamiliar with mathematics, and to such people the laws of intensity must inevitably remain a rigmarole of nonsense.

When two things vary together—

(*a*) One may vary *directly* as the other.

This is the simplest case of all. If you earn £1 per day, you earn £2 in two days, £3 in three days, and so on. In mathematical language the facts are stated in the formula that you earn £n in n days, where n stands for any number you like to say.

When £1 per day is the wage we say that the unit of time is a day, the unit of money a sovereign; and if we know the amount paid in wages *or* the amount of time worked, we can find out the unknown from the known by the simplest mental arithmetic.

An instance of such variation in Acoustics occurs when we consider the effect of pressure on air. Take a tube which is open at one end only, and insert a piston in that opening. Fig. 13 *a* represents this tube and piston in a state of rest—i. e. when the atmospheric pressure is exactly equal on each side of the piston.

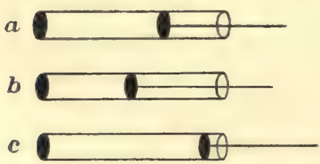


FIG. 13

In fig. 13 *b* the piston has been forced down the tube, and though the atmospheric pressure on it from outside (i. e. from right to left) is the same, the pressure of the air from inside is greater than normal. The air in the enclosed chamber has been forcibly compressed (i. e. its density has been increased) and a corresponding force is required to keep the piston in position—that is, to prevent the elasticity of the air from forcing the piston back to its normal position in fig. 13 *a*.

In fig. 13 *c* the piston has been moved by force in the opposite direction, and the air in the enclosed chamber has been diminished in density. If we remove the force which holds the piston in this position, then the pressure of the atmosphere at the free end (which is still normal) will force the piston back again to the normal position of fig. 13 *a*; and this means that the pressure of the enclosed air, which rose above normal when the density was increased, falls below normal when the density is diminished.

This fact is embodied in Mariotte's Law, that the pressure of a mass of air varies as its density, so that when the pressure is doubled the density is doubled also.

(*b*) One may vary *inversely* as the other.

A number is inverted when it is turned upside down. Thus the inversion of $\frac{3}{4}$ is $\frac{4}{3}$: the inversion of 5 (which is equivalent to $\frac{5}{1}$) is $\frac{1}{5}$.

If a man can dig a plot of a certain size in 12 hours, he will take 24 hours to dig two such plots, 36 hours to dig three, and so on; and the variation, as between time spent and work done, is direct.

But if two men set to work on the original plot the digging will take six hours only; if three men, then four hours, and so on. And in this case we say that the time taken varies inversely as the number of the workers. If you double the labour you halve the time: *multiply* the workers by any number you like and you must *divide* the time by the same number. Or, in mathematical language, n workers will finish the job in $\left(\frac{1}{n}\right)^{\text{th}}$ the time.

An example of such variation will be found, later on, to exist between the vibration-numbers of notes and the lengths of pipes or strings required to produce them.

(*c*) One may vary *directly as the square* of the other.

The grasp of the above depends on the recognition of one simple geometrical fact illustrated in fig. 14. Draw an acute angle HAK, and bisect it by a dotted line. Then measure off a series of equidistant points along the dotted line. In fig. 14 the four points so measured (p^1, p^2, p^3, p^4) are all (let us suppose) an inch apart.

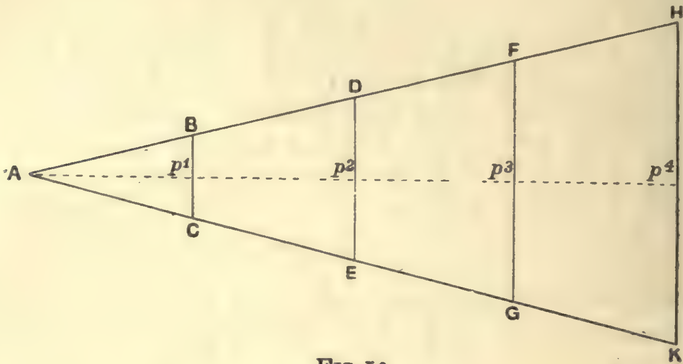


FIG. 14

If we now draw, through these points, lines at right angles to the dotted line, and limited on each side of it by the lines AH and AK (BC, DE, FG, HK), then such lines have an elementary geometrical relationship. For DE, being exactly twice as far from A as BC is, is exactly twice the length. FG is, similarly, *three* times the length of BC, and HK four times.

Let us now imagine that a lantern is made to shine on a white sheet in a dark room—just the ordinary magic-lantern of a village entertainment. If we insert a slide which is quite black except for a small square in the middle, then a square of white light will be thrown on the sheet. Assuming that the lantern is shining at L (fig. 15), then BCDE represents the square of light; and if BC is exactly one yard, then there is one square yard illuminated on the sheet.

FIG. 15

Now if we move the lantern *twice* as far away from the sheet, we know (from fig. 14) that BC will become *twice* as long.

That is to say, the square will now have a side 2 yards in length and *four* square yards in area. Similarly, if we move the lantern *three* times as far we get a square whose area is 9 square yards, *four* times as far giving us 16 square yards, and so on.

We can tabulate these results as follows:

1 unit of distance gives us an area of 1 square yard.

2 units of distance give us an area of 4 square yards (2^2).

3 units of distance give us an area of 9 square yards (3^2).

4 units of distance give us an area of 16 square yards (4^2).

I. e. the area of light on the sheet varies directly as the square of the distance.

Two applications of this form of variation to Acoustics may be pointed out here:

(1) When the density is the same, the velocity of sound in one medium compared to its velocity in another varies directly as the square root of the elasticity. If one medium is four times as elastic as the other, the velocity of sound in it is twice as fast; if nine times as elastic, then three times as fast.

(2) The first Law of Intensity, discussed in the next chapter.

(*d*) One may vary *inversely as the square* of the other.

The same example of the magic-lantern will serve to illustrate this form of variation if, instead of the *area* illuminated, we consider the *intensity* of illumination. The same amount of light which, at one unit of distance, had to fill a space of 1 square yard of sheet is obliged, at twice the distance, to spread itself over 4 square yards; and consequently it will only be a quarter as strong at any point. At three units of distance it has to cover 9 square yards, and will be $\frac{1}{9}$ as strong.

Thus we can say that the strength, or intensity, of illumination varies inversely as the square of the distance between the lantern and the sheet.

As acoustical examples of such variation we may add:

(1) When the elasticity is the same, the velocity of sound in one

medium compared to its velocity in another varies inversely as the square root of the density. Oxygen and hydrogen, for example, have the same elasticity, but the former is 16 times as heavy as the latter (i. e. its density is 16 times as great). Therefore sound travels in oxygen only $\frac{1}{4}$ as fast as in hydrogen.

For those who are not dismayed by mathematical formulae the above law may be expressed as follows:

If the elasticity (E) *and density* (D) *of two different media are known, the comparative velocity of sound* (V) *in them may be calculated from the formula*

$$V = \sqrt{\frac{E}{D}}.$$

(2) The time-period of the swing of a pendulum varies inversely as the square root of its length. If you multiply the length by four the number of oscillations is half as many; multiply the length by nine and the number of oscillations is one-third.

(3) The second Law of Intensity, discussed in the next chapter.

CHAPTER VII

INTENSITY

INTENSITY is the second of the three elements (Pitch, Intensity, and Quality) into which we can divide any sound, and it is merely another name for 'loudness'. We have seen that it is possible to make a vibrating body register its vibration in black and white, and that the wave-curves so registered have three characteristics—length, amplitude, and shape. And we saw, further, that pitch depended solely on the pace of the vibrations: i.e. on the length of the wave, since the slower the tuning-fork in fig. 2 (p. 23) vibrates, the fewer wave-lengths will it trace on the smoked glass.

Intensity depends on the *amplitude* of the vibrations, and on nothing else.

When we see a body agitated into motion by some external stimulus—a spinning-top or a plucked violin-string will serve as illustrations—we know by experience that as time passes the agitation imparted to the body by the stimulus decreases; i.e. unless we renew the stimulus the movement will gradually diminish and ultimately come to an end. We suspect this fact, and learn to rely on it, many years before we know anything about friction or elasticity. Similarly we know that a tongue of metal, fixed in a vice and made to vibrate, will at length reach its position of rest.

Now the distance from the position of rest to the extreme point of any oscillation is called, as we know, its amplitude, and this amplitude gradually decreases throughout the whole process of vibration. But we also know, by experience, that the sound, of which the vibrating tongue is the source, grows at the same time gradually weaker and ultimately ceases.

And our natural suspicion that these two diminishing quantities—created at the same moment, simultaneously at the full, decreasing together and disappearing at one and the same instant—must be interdependent and causally connected, is true, and is founded on physical law.

The student should be quite clear that the amplitude of the curve in fig. 2 (p. 23) depends solely on the distance which the agitated prong swings away from its position of rest. As the oscillations of the prong decrease from their maximum to zero, so the amplitude of the curve decreases from its fullest width to nil—since, when the fork is at rest, the needle will trace a straight line on the moving glass.

By careful experiments physicists have discovered that there is a definite connexion between amplitude and intensity, and have established the law that intensity depends on amplitude alone.

The two laws of Intensity are as follows:

(1) Intensity varies directly as the square of the amplitude of Vibration.

If two points be taken on the curve traced by the tuning-fork on smoked glass such that the amplitude of one is twice the amplitude of the other, then the volume of sound at the moment of tracing the curve of greater amplitude was *four* times that at the moment of lesser. In other words, when the swing of the prong or fixed metal tongue is doubled the resultant sound is multiplied in volume by 4.

(2) Intensity varies inversely as the square of the distance from the vibrating body.

If you are listening to the sound of a trumpet at a distance of 100 yards, and then walk away from it until the distance is 200 yards, you will hear only a *quarter* of the volume of sound that you heard in the first instance; while at 300 yards you will hear only a volume of one-ninth.

The second of the above laws may perhaps be more easily grasped if it is connected with the earlier illustration of the magic-lantern.

Areas of spheres vary as the squares of their radii. That is to say, if a round football-bladder of given radius is blown out until the radius is *doubled*, it will require *four* times the amount of leather to cover it. And since unimpeded vibrations spread out equally in all directions from their source (like an expanding bladder), we can see that the

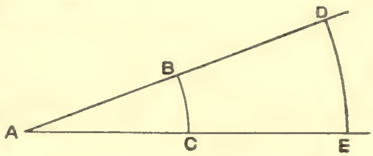


FIG. 16

vibrations starting at A (fig. 16), which have to cover an area whose side, when the radius is AC, is the arc BC, will have to cover an area whose side is the arc DE when the radius has been doubled into AE. And the latter area is four times the former.

It must be remembered that the Intensity of sound, though following the above laws when conditions are ideal and constant, is interfered with to a certain extent by several fortuitous and more or less incalculable circumstances. Thus the direction and power of the wind, and also the density of the air at a given moment, will modify the volume of sound. The influence of wind, though very great, cannot be calculated with any approach to exactness; and the question of the variations in density of our atmosphere from time to time does not fall within the scope of knowledge essential for a musician.

But Intensity is enormously affected by one other consideration into which it is imperative we should inquire, viz. the presence, accidental or otherwise, of some body which, by *sympathetic* vibration, will reinforce the vibrations of the original body. The next chapter will deal with reinforcement of this kind, which is called Resonance.

omit

CHAPTER VIII

RESONANCE

When the vibrations of the air come into contact with an obstacle which prevents their normal progress, any one of three things may happen.

They may be (1) reflected,
(2) destroyed,
(3) refracted.

(1) **Reflection** occurs when, owing to the hardness of the obstacle, the vibrations rebound and continue their course in a changed direction. It is clear from this that if we take up a position in a building and listen to a singer or speaker, we may actually receive through our ears not only the air-vibrations which come to us direct from his mouth, but also reinforcing vibrations which are reflected from various parts of the walls and ceiling. When these auxiliary vibrations reach us simultaneously with the direct ones—or at so nearly the same instant that the combination results in one reinforced sound—we say that the resonance of the hall is good. When confusion results the local reporter will repeat what every one else has been saying—'the acoustical properties of the building are unsatisfactory'.

When reflection occurs in an exactly opposite direction—as when we sing a note down a straight corridor, or across the water towards a perpendicular cliff—it creates a thing familiar to every one, i. e. Echo.

(2) **Destruction** occurs when the obstacle is so soft in substance that the natural resilience, or power of 'bounce', in the air (due to its elasticity) is prevented from coming into play.

A similar result follows if we throw a tennis-ball against a feather mattress.

The practical importance of this fact is great. For instance, it gives an adequate explanation of the well-known fact that a room is less resonant when full of people than when empty; for the soft material of the clothes of the audience is engaged in killing all vibrations which come in contact with it. The resonance of the room would not be diminished if the audience were replaced by an equal number of undraped stone statues.

An interesting case of the intentional use of a soft obstacle is provided by the chapel of Keble College, Oxford. This building is not only high in comparison with its length and width, but also has walls which are hard and smooth, and of an almost unbroken surface. When it was first used it was found to be so full of resonance and echo that a speaker's words became a mere jumble of sound. To remedy this a large curtain was hung across the west end which, by destroying vibrations that would otherwise have been reflected, very largely improved the conditions of hearing.

(3) **Refraction,** since it refers to the change (or 'bending') of the direction of vibrations due to conditions in the air itself, will not be dealt with until we approach the whole question of Transmission.

The meaning attached to the word Resonance up to this point is the one which most human beings would, in conversation, understand it to convey. It is not, however, the scientific meaning of the word. Scientifically Resonance refers to that increase or reinforcement which a sound can acquire through the co-operation of other vibrating bodies or columns of air, whose auxiliary vibrations will add something more to the sum-total of sound resulting from the original body alone.

Resonance proper is of three kinds:

(1) The original vibration may be reinforced (i.e. the amplitude of the vibration may be increased, and the resulting tone thereby made louder) by the help of some other body

which, whatever its shape and size, will vibrate at the same rate.

(2) We can secure the same result by deliberately bringing the original body into contact with a column of air whose length has a calculated relationship to the wave-length of the original vibrations.

(3) We can induce the sympathetic vibration of a second body by a stimulus conveyed to it through an intervening air-space.

In these three cases the reinforcement is obtained

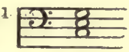
- in (1) by the direct impact of a vibrating body on a body at rest,
- in (2) by the action of a vibrating body directly on the air,
- in (3) by the excitement communicated from one body to another by means of air-vibrations.

(1) An example of this kind is seen when we place the end of a vibrating tuning-fork on a table. The wood of the table, independently of its dimensions and shape, is agitated into a state of vibration which, in the matter of rapidity, is under the control of the fork. The sound of the fork, ordinarily so feeble as to be inaudible unless the instrument is placed quite close to the ear, now becomes so strong that it can be heard plainly at a distance of many yards.

(2) A simple example of direct action on the air by a vibrating body is provided by the ordinary organ-reed. Such a reed produces by itself a poor and somewhat raucous tone, whose pitch is governed by the pace at which the reed vibrates. Knowing this pitch we can, of course, find its vibration-number; and knowing the vibration-number, we can find the wave-length of the sound in air. Constructing a pipe of such a length (i. e. a pipe containing a column of air of the length required), we place it over the reed, and the column will 'catch' the vibration, and by vibrating in sympathy will so reinforce the sound that it attains a full and rich quality.

Another example is found in the human voice, where the comparatively weak vibrations of the vocal cords are reinforced by the columns of air in the mouth, throat, and nasal cavities. One of the main objects of voice-training is to secure, by practice, such instinctive muscular control of these cavities that the air enclosed in them is always of the exact dimensions required for the reinforcement of the note which the vocal cords are producing.

(3) Many simple experiments may be made by any one who wishes to establish the principles of the third and commonest form of Resonance.

(*a*) Silently depress on a piano the three notes[1] .
Then sing *into* the piano, fairly loudly, any of the three notes. As a result the strings corresponding to whichever note you sing will at once begin to vibrate in sympathy, and when you cease singing the piano will continue (as long as you keep the dampers off the strings by holding the notes down) sustaining whichever note you have sung. That is to say, the piano-strings, having been stretched to that definite degree of tautness that will enable them to produce a certain definite number of vibrations per second, can produce them either directly, when acted on by a hammer, or sympathetically, owing to the fact that the air around them is already vibrating at the same pace as that at which they themselves cause it to vibrate.

(*b*) Take an open pipe between 2 and 4 feet in length. Such a pipe, if used as an organ-pipe, would produce a note somewhere between tenor C and middle C (*c* and *c′*). If you then sing slowly up or down within these limits with your mouth close to one end of the pipe, you will find that suddenly one note acquires an enormously increased resonance; and this is the note which is 'proper' to the pipe.

Most people—at all events those old enough to recall a time

[1] Each experimenter must, naturally, choose a chord within the range of his voice.

when the electric light was not common—will have noticed a practical example of this kind of resonance, when the vibration of a gas-flame happens to coincide with the note proper to the glass tube or globe around it. A shrill and unpleasant note of definite pitch is the result, and we destroy it by turning the gas either up or down—i. e. by altering the rate of vibration of the flame.

(*c*) Take two tuning-forks (preferably mounted on sounding-boards) in exact unison; set one of them in motion and place it near the one at rest. Gradually the silent one will begin vibrating, and you will hear its note after you have damped the original fork into silence. If you alter the vibration-rate of the second fork by attaching a pellet of wax—even to the small extent of making its frequency 262 instead of 264—it will fail to sympathize.

In all such experiments it is necessary to wait a little for sympathetic vibration. The air-vibrations have to produce vibration in a body at rest by continually hammering at it, and as the effect of a single such vibration is almost nothing it is only the accumulated effect that tells in the long run. If we place a very heavy weight in a swing, and tell a small child to set it swinging, the child may say he cannot move it; but if he can move it by the smallest fraction of an inch he can ultimately get it into full swing (if his strength and patience hold out), and his feat is, like that of the air-vibrations on the silent fork, an illustration of what is called *cumulative impetus*.

(*d*) Tune the two lowest strings of a violoncello in unison. If you then pluck the C string fairly sharply it is quite easy to see with the naked eye its neighbour agitating itself; and if you damp the C string the instrument continues to produce the same note quietly, owing to the vibrations communicated to the string that was not plucked. When the instrument is lying flat it is interesting to place a small piece of paper, bent into Λ-shape, on the G string. Almost as soon as the C string is plucked the paper will jump off the G string; whereas it will lie peacefully at rest on either of the two top strings.

From the above and similar experiments the two following laws are formulated:

(1) Maximum resonance results when the two bodies concerned are in exact unison; i.e. when the sound proper to each of them has exactly the same vibration-number.

(2) Resonance does not occur in such cases immediately, since a short period is required by the original vibrating body (or the air-vibrations caused by it) in which to excite the sympathetic vibration of another body by cumulative impetus.

The immense practical importance of the principles of Resonance will be realized by any one who considers the fact that all musical instruments are merely ingenious ways of securing sympathetic vibration. In a later chapter the methods and differences of instruments are inquired into in detail, but it may be pointed out here that even in such well-known instruments as the piano and violin the *tone* is due entirely to the reinforcement secured by sounding-boards and belly. We might remove all the wood-work from a piano, leaving the bare action and the iron frame with the strings attached; or we might stretch four violin-strings to their usual tension, by means of pulleys and weights, without using any enclosed chamber; but in neither case could we, in the absence of the means of reinforcement, produce any sound of the slightest musical value.

In making experiments we sometimes require to know, when we are listening to a collection of sounds, whether one particular note is present. To enable us to do this with certainty Helmholtz invented a device called a *Resonator*.

These are spherical and hollow globes of metal or glass with two holes, one at either end of a diameter. One hole is small, and shaped to fit the ear; the other is larger, and is for collecting the vibrations of the air corresponding to the note of the resonator (see fig. 17). If you are listening to a collection of sounds and wish to know for certain whether middle C is present, you take the resonator tuned for that note, insert the small end in one ear, and close the other ear with your hand. If the note is not present nothing will happen; but if it should prove to be there the resonator will soon vibrate so vehemently that scarcely anything but that one note can be heard.

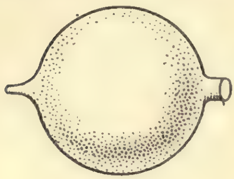


FIG. 17

It must be noticed that in the case of resonators the sympathy is practically immediate; for the vibrations already in the air act at once on the air enclosed, and do not have to wait until the metal or glass is in a state of vibration.

May 9, '19

PART IV. QUALITY

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CHAPTER IX

TYPES OF MUSICAL TONE

In an earlier chapter we described (p. 25) the six methods resorted to for the purpose of producing periodic vibration. These methods will now be considered with regard to the actual process by which each of them originates the air-vibrations which result in the various types of tone at the disposal of musicians. For this purpose we can consider the six classes of instruments as being:

(1) Flue-pipes.
(2) Stringed instruments.
(3) Reed instruments.
(4) The voice (a form of reed).
(5) Drums (percussion instruments with membranes).
(6) Bells (solid percussion instruments).

(1) **Flue-pipes.** Every one knows the construction of the penny-whistle. A column of air is forced through a narrow channel (the mouthpiece); it strikes an edge of metal (the 'lip') placed in such a manner as to disturb the unity and

Fig. 18

direction of the advancing column. The agitated air then begins to vibrate at many different rates, from which the pipe selects the one rate which, by virtue of its length, it is qualified to reinforce. When all the holes in the pipe are closed by fingers we can, knowing the pipe-length and the velocity of sound, prophesy the pitch; and when the hole nearest the open end is freed by lifting a finger we merely have to deal with a pipe of shorter length.

The flue-pipe of an organ is simply, in principle, a penny-whistle in a vertical position. In order that it shall stand upright the 'mouthpiece', through which the column of air is to be forced, is placed in line with the horizontal diameter of the tube; but the air is directed through a narrow slit on to the 'lip' in exactly the same way (fig. 19).

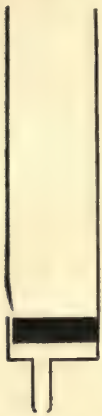

FIG. 19

The vibration-fraction of the note produced by an open pipe is found by dividing the velocity of sound (say 1,100 feet per second) by twice the number of feet in the length of the pipe. The reasons for this will be discussed later; at this point the student may accept the statement that if the open pipe in fig. 19 is 4 feet in length, its note will have, as its vibration-number,

$$\frac{1100}{8} = 137{\cdot}5.$$

For stopped pipes we divide by four times the length.

N.B. *The pitch of a flue-pipe sharpens as the temperature rises.*

A rise in temperature expands the air—i. e. lessens its density. Then the velocity of sound increases. Consequently the fraction

$$\frac{\text{velocity of sound}}{\text{twice pipe-length}}$$

must (if the pipe-length remains unaltered) grow greater as the numerator increases. That is to say, the number of vibrations per second produced by the pipe must increase (i. e. its pitch must sharpen) as the temperature rises. As the expansion of wood under any ordinary rise of temperature is negligible we may consider the pipe-length as constant; metal expands more, but not enough to counteract the effect of air-expansion.

(2) **Strings** may be made to vibrate in three ways: by bowing (violin), striking (piano), or plucking (guitar).

When a stretched string is excited into a state of vibration it appears to become much thicker towards the middle than at the fixed ends; and this is because its swing is of the same nature as the swing of a skipping-rope. If you place two people (B and C) at equal distances from yourself (A) and a few feet away from each other—so that the three of you form an isosceles triangle with yourself at the vertex (fig. 20)—then a skipping-rope turned rapidly by them will present to you the appearance of the loop B C, which is a vibration-form with fixed ends (called by scientists a stationary vibration). Its form, to your eye, is something of two dimensions in one plane, and can be measured as so long from right to left, so high from top to bottom. But the actual particles forming the rope are describing paths in a series of planes at right angles to the plane of the figure which you see. This should be clear if you imagine a piece of ribbon tied to any point of the rope; for it will clearly travel in a circle *to* and *from* you.

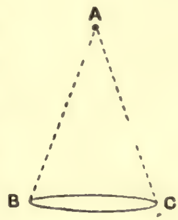

FIG. 20

A stretched string vibrates in the same way, and its vibrations are called *transverse* vibrations because the movement between the fixed ends is really made by particles moving at right angles to the string-length. This name distinguishes them in character from *longitudinal* vibrations (such as those of the air), in which the particles oscillate in the direction in which the vibration is travelling.

Transverse vibrations obey the laws of pendulum motion. A guitar-string is plucked away from its position of rest, and immediately starts trying to regain that position; and unless it is plucked a second time it must succeed in doing so. This gradual diminution of amplitude, and consequent decrease of intensity, is avoided in the violin, since the bow-pressure, which can be increased or decreased at will, regulates the amplitude of vibration.

Four considerations govern the pitch of a stretched string:

(*a*) its length,
(*b*) its diameter,
(*c*) its tension,
(*d*) its density.

The effects of these four factors are expressed in the following four laws:

(*a*) *The vibration-number varies inversely as the length of the string.*

E. g. Double the length of string and you will halve the vibration-number; that is, the resulting note will be an octave lower.

(*b*) *The vibration-number varies inversely as the diameter.*

E. g. Halve the diameter and you will double the vibration-number; that is, the resulting note will be an octave higher.

(*c*) *The vibration-number varies directly as the square root of the tension.*

E. g. Screw the string four times as tight and you double the vibration-number.

(*d*) *The vibration-number varies inversely as the square root of the density.*

E. g. Use a string of four times the usual density, and the vibration-number of the note it gives will be *half* that of the usual note.

N.B. *Stringed instruments fall in pitch when the temperature rises.* The increase in the velocity of sound is more than counterbalanced by the fact that strings expand with heat and consequently lose their tension.

(3) **Reed Instruments.** In fig. 21 we have an oblong strip of wood or thick material (A B C D), out of which a smaller oblong (E F G H) has been cut. If we cut a piece of elastic

metal to a size a little longer than E F and slightly broader than E H, and then screw it at K over the opening E F G H, we can so cover that opening that wind blown from above on to the metal strip will not pass through, but will rather close the passage more tightly in proportion as the force of the wind is increased.

If, however, the wind is blown laterally in the direction of the lines F E and G H, instead of from above on to those lines, it will insinuate itself between the metal and the wood, lift the strip, and pass through the opening. But the elasticity of the strip will cause it to rebound into its original position and cut off the passage of air through the opening; and then the same process will repeat itself, causing a series of puffs in the air, whose frequency depends on the quickness of movement of the strip that blocks the passage.

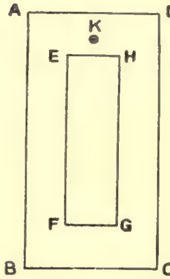

FIG. 21

This is an example of a 'beating' reed, and we can control its action in three ways, each concerned with one of the three essential qualities of a sound:

(1) By regulating the tightness of the screw, by selecting a metal of the required elasticity, and by altering the length of the tongue, we can secure vibrations in the air of any given rapidity; i. e. we can control the pitch.

(2) By regulating the force of wind we can alter the displacement of the tongue—which means the amplitude of its swing; i. e. we can control the intensity.

(3) By the application of pipes or chambers containing columns of air of varying shapes and dimensions corresponding to the pitch chosen we can reinforce the tone; i. e. we can, within limits, control the quality.

When the tongue of metal, which is the 'reed' itself, is cut so that it will just pass through the passage, we have an example of a 'free' reed. One puff will pass when the reed is elevated, being cut off when the reed rebounds to its normal position; but then the reed, instead of having to repeat its action, will continue its swing *through* the opening, let another puff pass, and regain its position of rest.

Many orchestral instruments are played by means of reeds. The clarinet is a beating reed, since the vibrations are made by forcing air between a thin wooden reed and a fixed mouthpiece with an opening smaller than the reed. The oboe and bassoon are double-reeds, because in them the vibrations are caused by forcing air between two reeds so placed that their edges meet—just as children often force air between two sheets of paper or two pages of a book. But in all the above instruments the pitch is governed by the length of the pipe attached.

In the case of brass instruments played with a cup-shaped mouthpiece the player's lips form the reed and the brass tube, as before, controls the pitch.

In the case of the human voice the vocal cords form a double-reed, and their tension governs the pitch.

N.B. *Metal reeds will flatten when the temperature rises*, the expansion of the metal causing a loss of elasticity (and consequent diminution of rate of vibration) which more than counterbalances the increased velocity of sound.

Little need be said of the two last types of tone. Elastic membranes, such as drums, produce notes whose frequency is controlled by the tension of the membrane; and the vibrations of elastic solid bodies, such as bells, lead to intricate and baffling questions which are the special work of campanologists.

[illegible] Ste
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tical work!

CHAPTER X

ON MATHEMATICAL PROGRESSIONS

In its early stages mathematics deals with definite numbers and quantities, known and unknown; but as it becomes more advanced it has to deal, to an ever-increasing extent, with groups of numbers arranged in certain orders. The technical name for these groups is 'Series'; and a Series is never a haphazard collection of numbers (or 'terms'), but always an intentional and logical arrangement according to some avowed plan.

The three simplest forms of series are known as the Arithmetical, Harmonic, and Geometrical Progressions.

I. Arithmetical Progression.

A series of terms is said to be in A. P. when each term differs from its predecessor by the same amount.

The following are three series:

(1) 1, 2, 3, 4, 5, 6, ...

(2) 2, 4, 6, 8, 10, 12, ...

(3) 3, 6, 9, 12, 15, 18, ...

All the above are Arithmetical Progressions, and every reader will know what is the seventh term in each case.

If all series were as simple as the above it would be possible to work out most questions in one's head; for, however 'bad at mathematics' a person may be, he will probably be able to say what would be the tenth term in any of the three progressions given. But not every one would give off-hand the tenth term of the A. P. whose first two terms are 57, 121, ...

A very small amount of algebraical knowledge will make all such questions easy.

If we call the *first* term of an A. P. a, and the common difference between the terms d (the common difference is obtained by subtracting any term from the next following), it will be seen that all arithmetical progressions take the following form :

$$a,\ (a+d),\ (a+2d),\ (a+3d)\ldots$$

That is to say, the tenth term *must* be $(a+9d)$, the twenty-fifth term $(a+24d)$, and so on.

This fact is expressed algebraically by saying that the n^{th} term (n merely means *any number you like to say*) of an A. P. is always

$$a+(n-1)\,d.$$

Thus the tenth term of the series beginning 57, 121, ... is

$$57+(9\times 64).$$

Any reader who can do easy mental arithmetic can become familiar with A. P. by setting himself easy problems at odd moments. E. g. What is the seventeenth term of the series 2, 5, 8, 11 . . . —? The answer is :

$$2+(16\times 3)=50.$$

II. Harmonic Progression.

When a fraction is turned upside-down it is—as was pointed out in discussing Variation—*inverted*; and a whole number may always be treated as a fraction whose denominator is 1. Thus, if we wish to invert 10, we call it $\frac{10}{1}$, and its inversion is $\frac{1}{10}$.

An H. P. is simply a series which, when every term is inverted, becomes an A. P.

The following two series are Harmonic Progressions :

(1) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

(2) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \frac{1}{14}, \ldots$

To discover the n^{th} term of such a progression we must invert the first two terms. Treating these as an A. P. we can get

10 = 1 × 3

the first term and the common difference (a and d) from which we find the n^{th} term of the A. P.

Invert this term, and you have the n^{th} term of the H. P.

E. g. Find the ninth term of the H. P. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11} \ldots$
This series inverted becomes the A. P. $2, 5, 8, 11 \ldots$
The ninth term of this is $2 + (8 \times 3) = 26$.
Therefore the ninth term of the H. P. is $\frac{1}{26}$.

III. Geometrical Progression.

Just as the terms in an A. P. differ by the continual *addition* of a given number, so the terms in a G. P. differ through the continual *multiplication* by a given number; and this latter is called the common *factor* (f).

The two following series form Geometrical Progressions:

(1) $1, 2, 4, 8, 16, 32, \ldots\ldots (f = 2)$.
(2) $2, 6, 18, 54, \ldots\ldots\ldots\ldots (f = 3)$.

Examining (2) we find that
The first term $= 2$
The second term $= 2f\ (2 \times 3 = 6)$.
The third term $= 2f^2\ (2 \times 3^2 = 18)$.
The fourth term $= 2f^3\ (2 \times 3^3 = 54)$.

From this it is easy to see that the tenth term will be (2×3^9), a fact which is expressed algebraically by saying that the

$$n^{\text{th}} \text{ term} = a \times f^{n-1}.$$

It is, of course, far more difficult to work out problems in G. P. mentally than those in A. P., but easy ones should be tried.

E. g. Find the ninth term of the following G. P.:

$$1, 2, 4, 8, \ldots$$

The ninth term must be af^8—i. e. 1×2^8—and this will prove to be 256, a number familiar to students of Acoustics as the Philosophic Pitch of middle C.

It is absolutely essential that the student who wishes to grasp the elementary facts of the Harmonic Chord, to say nothing of the more difficult question of Temperament, should have in his mind a clear and distinct conception of the fundamental characteristics of the three Progressions explained above.

If they still present any difficulties to his mind he must either wrestle further with them, or apply to some more learned friend for their elucidation; for his whole grasp of the foundations and structure of Acoustics from this point onwards depends on his apprehension of the principles underlying these three Series.

May 1.

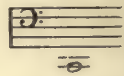
CHAPTER XI

PARTIAL TONES

Up to this point we have treated musical sounds as if each one were a simple, self-contained, isolated phenomenon. When, for instance, the note known as 'middle C' has been mentioned, we have assumed (though the assumption has not affected the truth of any conclusion arrived at) that when a sounding-body vibrated at a certain pace the resulting vibrations communicated to the air would give us the sound of middle C and nothing else.

It is, indeed, possible to produce such a pure musical sound, and when this is done we call it a 'simple' sound, and say that it is produced by 'simple' vibrations. But just as few of the colours which meet the eye in the course of a day are due to light of one kind only, the great majority being due to combinations of many kinds of light, so almost every sound we hear, of whatever pitch and quality, is in reality a combination of simple tones of different pitches, manipulated by the ear so as to give the impression of a single sound.

The above statement does not refer (like Browning's 'star') to the combinations of sounds produced by different bodies, or by a body capable of producing simultaneous sounds; but to something more subtle than the mere ability of the mind to apprehend a chord. It means that when we are apparently listening to one single sound of definite pitch, such as a single note struck on the piano, we are almost invariably in reality listening to a combination of sounds of different pitches which sum themselves up into one resultant sound. If we strike B flat or C sharp on the piano we think, until we know better, that we hear the sound of the particular note struck, and no other sound; but a very little ear-training will soon convince the most sceptical that the one note apparently heard in isolation is only *part* of what is audible, and that various other sounds of a higher pitch are included in it.

Almost every one, when confronted for the first time with the above statement of fact, is inclined to doubt its truth. Until convinced by practical illustration he will look on it as an ingenious theoretical hypothesis, since no one willingly admits that he has, for a life-time, been deceived by the evidence of one of his own senses. Such a person should, at this point, make a practice of continually going to a piano and striking (firmly and loudly) the note , listening to it with concentrated and patient attention, with the one idea of detecting other sounds than C. Sooner or later will inevitably come the moment when the sound of G (g), the twelfth above, fills his ears with such persistence that it will seem incredible that up to then the note was unnoticed. After this moment the recognition of other sounds is merely a matter of industry.

The ordinary musical sound, then, is not a pure or simple tone, but a complex sound, compounded of a series of notes of different pitch each of which, when we isolate it (by resonators, &c.), proves to be in itself a pure tone.[1] This series consists of a fundamental note plus its overtones, or harmonics; but it is essential, for reasons which will appear later, that the student should not use the terms 'overtones' and 'harmonics' except as group-names convenient in conversation. In all acoustical investigation the whole series is said to consist of Partial Tones, the fundamental note (or

[1] It is sometimes stated that Helmholtz called such a compound sound a *Klang*, and the corresponding English word *Clang* has come into use for the same purpose. But as a matter of fact this is not true. Helmholtz distinguished Klang (= musical sound) from Geräusch (= noise)—[*Tonempfindungen*, p. 14]—and later on uses Klangfarbe for Quality.

Prime Tone) being termed the First Partial, the first overtone the Second Partial, and so on.

In fig. 22 we have a chord of eight notes, sometimes called the Harmonic Chord, which gives us the first eight partial tones of the note C. The numbers at the side, forming an elementary arithmetical progression from 1 to 8, give us at once the *number* of the particular partial tone. E (e′) is the fifth partial, B flat (which, it may be noted, is not in tune) is the seventh partial.

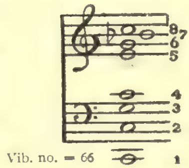


FIG. 22.

The numbers also disclose another important fact; for, if we take the geometrical progression 1, 2, 4, 8, &c., we find that the partials corresponding to these numbers are always C; and we might guess that if we explored amongst still higher partials (for fig. 22 is only the lowest part of the harmonic chord, which goes on upwards to the very limits of audibility) we should find that all the terms of this geometrical progression would be the numbers of partials representing C. This guess would prove to be right, for the sixteenth, thirty-second, sixty-fourth, &c., partials are C also, each C being an octave higher than the last.

Thus, if you have to construct a table of partial tones from a given note B flat, the first partial is the note given, the second is an octave higher, the next octave is the fourth, the next the eighth, and so on. And the same law of geometrical progression applies to *all* partials; for in fig. 22 G (g) is the third partial, its octave is the sixth, the next octave will be the twelfth, the next the twenty-fourth, &c. And since the fifth partial is E (e′), the tenth, twentieth, fortieth, will all prove to be E.

The Harmonic Chord of fig. 22 also enables us, by the application of the progressions, to find the vibration-number of any partial if the frequency of the prime tone is known. For the vibration-number of the third partial is three times that of the prime tone; that of the seventh is seven times that of the prime tone. Thus, given the vibration-number of the prime tone of fig. 22 as 66, then the vibration-number of e', the fifth partial, is

$$66 \times 5 = 330.$$

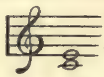
Again, from fig. 22 we can discover the vibration-fraction of intervals.

A vibration-fraction is a modulus which enables us to find the vibration-number of one note of an interval if we know that of the other. If one note has a frequency of 200 and another has 150, then the vibration-fraction of the interval they form is

$$\frac{200}{150} = \frac{4}{3}.$$

If we were told that two other notes formed the same interval, and the vibration-number of the lower note was 66, then we know the frequency of the higher note is

$$66 \times \frac{4}{3} = 88.$$

Suppose we wish to find the vibration-fraction of a major third. In fig. 22 there is a major third , and we know that the frequency of the upper note is 330 (i. e. 66×5); that of the lower note 264 (i. e. 66×4). So the vibration-fraction is $\frac{330}{264} = \frac{5}{4}$.

The sum worked out merely proves that the vibration-fraction of the interval between any two notes in fig. 22 is the fraction formed by the numbers belonging to each note as partials. Thus the vibration-fraction of a minor third is $\frac{6}{5}$; that of a major sixth is $\frac{5}{3}$.

In this way we can find, from fig. 22, the vibration-fraction of

octave,
major and minor sixth,
fifth,
fourth,
major and minor third,
unison.

If we can find the fractions belonging to major and minor seconds and sevenths we can compile a list covering the octave;

and these four intervals can be found quite easily by means of a little ordinary arithmetic.

Major Seventh.

We know the frequency of the sixth partial, G (g′), for it is six times that of the prime tone, and

$$66 \times 6 = 396.$$

We know the vibration-fraction of a major third is $\frac{5}{4}$; therefore the frequency of B (b′), the major third above the sixth partial, is

$$396 \times \tfrac{5}{4} = 495.$$

Now the frequency of this note b′, divided by the frequency of middle C (c′), the major seventh below it, will give us the vibration-fraction of a major seventh:

$$\tfrac{495}{264} = \tfrac{15}{8}.$$

Minor Seventh.

From middle C (264) find the F (f′) above it, by means of the vibration-fraction of the fourth, $\frac{4}{3}$.

$$264 \times \tfrac{4}{3} = 352.$$

The G (g) below this f′ having a frequency 198, we get the vibration-fraction of a minor seventh as

$$\tfrac{352}{198} = \tfrac{16}{9}.$$

Major Second.

The same f′ (352), with the g′ above it (396), forms a major second. Therefore the vibration-fraction of a major second is

$$\tfrac{396}{352} = \tfrac{9}{8}.$$

Minor Second.

The same f′ (352), with the e′ below it (330), forms a minor second. Therefore the vibration-fraction of a minor second is

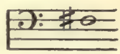
$$\tfrac{352}{330} = \tfrac{16}{15}.$$

We can now give a complete list of the vibration-fractions within the octave, omitting only the unnecessary tritone.

Octave = 2	Fourth = $\frac{4}{3}$
Ma. 7 = $\frac{15}{8}$	Ma. 3 = $\frac{5}{4}$
Mi. 7 = $\frac{16}{9}$	Mi. 3 = $\frac{6}{5}$
Ma. 6 = $\frac{5}{3}$	Ma. 2 = $\frac{9}{8}$
Mi. 6 = $\frac{8}{5}$	Mi. 2 = $\frac{16}{15}$
Fifth = $\frac{3}{2}$	Unison = 1

N.B.—Students are strongly urged not to attempt to commit the above table to memory. If it is borne in mind that, when the Harmonic Chord of fig. 22 is extended, the 9th partial is d″ and the 15th is b″, then any fraction can be instantly called to mind. The more roundabout methods just employed for finding the fractions of sevenths and seconds were used in order to show how such problems *can* be solved when the facts given are limited.

The facts dealt with in this chapter will be the better remembered, and the more believed in, if their truth is corroborated by simple experiments.

Put down the note F sharp  on the piano, without allowing it to sound. Then strike several times, sharply and loudly, bottom C, the prime tone of fig. 22.

When the C strings have been damped the result is silence, although the F sharp strings were perfectly free.

But try the same experiment again, holding g (the semitone above the F sharp), and you will find the note g sounding loudly; which means that there was a note of the same pitch already present in the C sound, which aroused the sympathetic vibration of the g strings.

The same experiment may be tried with firstly the E flat, and secondly the E natural (e′) above the g already tested; and the note which is a partial tone to C will be the one to vibrate in sympathy.

A similar test may be made by placing a small Λ-shaped piece of paper on the G string of a 'cello as it lies flat. When the C string is plucked nothing happens. But if you place a finger on the G string, reducing its length to that required for any partial tone of C, the paper will fly off the string as soon as the C string is sharply plucked. The same facts are established as in the previous experiment.

An ingenious instrument has been built by Mr. Rothwell, the well-known organ-builder, on which some experiments can be made with partial tones. A prime note (low G) is produced by a soft bourdon pipe, and its partials, up to the twenty-fifth, can be sounded in any combination desired (by wedging down the notes) on soft dulciana pipes. When the five lowest partials are sounded the result is simply a soft and pleasant chord of G major; but as other partials are added the sound of the chord gradually vanishes, whilst the prime tone advances into the foreground with ever-increasing volume. When all twenty-five partials are sounding together—each, be it remembered, quite soft by itself—the result is one enormous low G of the unmistakable quality of a trombone.

A curious phenomenon can be remarked on this instrument. When the ten lowest partials are sounding (i. e. the eight of fig. 22 with d″ and e″ added, the whole being transposed, since the prime tone of this particular instrument is not C but G) they seem, for the first time, to coalesce into one note instead of a chord. And this one note forms a perfectly satisfactory bass for *any* of the other notes which are sounded with it. If, for instance, the 15th partial (in this case F sharp) is sounded with the prime note G the result gives no impression whatever of a major seventh, but the G (without, of course, in any way changing its pitch or quality) seems to assimilate itself to the sound of F sharp in much the same way as the sound of a triangle assimilates itself to any note played with it.

In this instrument experiments, though extremely interesting, cannot be carried very far, partly because all the partials, though soft, are of comparatively the same strength, and partly because each of them has a series of partial tones of its own. If an organ of this kind were constructed in which care were taken to produce pure tones only, and to regulate their intensity, it would be possible to establish a good many conclusions as to the nature and effects of overtones.

There is one more group of important facts, the key to which is supplied by the Harmonic Chord of fig. 22, namely, the relative length of strings and pipes.

Up to this point we have been content with the knowledge that, other conditions being equal, shortening the length of a string or pipe raised the pitch of the note resulting from it, and have not asked whether any definite relationship could be established between the length and the pitch. Our knowledge has been, for the most part, qualitative without being quantitative.

The relationship we are seeking for is found by the application of harmonic progression to the series of partials. If we know that the length of string or pipe required to produce any given note is x feet, then the length required to produce its fifth partial is $\frac{x}{5}$, its fifteenth, $\frac{x}{15}$, and so on. Thus the length of pipe required for bottom C, the prime tone of fig. 22, is roughly 8 feet; the length required for d″, the ninth partial, will be

$$\tfrac{8}{9} \text{ feet} = 10\tfrac{2}{3} \text{ inches.}$$

If the A string of a 'cello is 2 feet long, then the length required for treble E (e″), the third partial, is

$$(2 \times \tfrac{1}{3}) \text{ feet} = 8 \text{ inches,}$$

which means that the finger must 'stop' the string at a point two-thirds of its length from the nut, and one-third from the bridge.

Vibration-fractions are used, in solving problems, exactly as in finding frequencies, always remembering that in harmonic progression the terms of a series are inverted.

For instance, suppose we are given the interval 

together with the vibration-number and length of string of the G, and are asked to find vibration-number and length of string of the E.

The vibration-fraction of a major sixth is $\frac{5}{3}$, and so the vibration-number of the upper note is

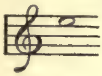
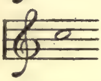
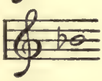
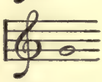
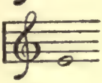
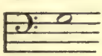
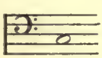
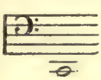
$$\text{Vibration-number of lower note} \times \tfrac{5}{3}.$$

But the length of string of the upper note is

$$\text{Length of string of lower note} \times \tfrac{3}{5}.$$

A table is appended of the first ten partials of C, with the elementary arithmetical and harmonic progressions necessary for ascertaining frequencies and string- or pipe-lengths.

TABLE OF THE FIRST 10 PARTIAL TONES, WITH FREQUENCY-MODULUS[1] AND STRING (OR PIPE) LENGTH-MODULUS.

Partials of C	Rank of Partial	Modulus for obtaining Frequency	Modulus for obtaining String and Pipe length
	10th	10	$\frac{1}{10}$
	9th	9	$\frac{1}{9}$
	8th	8	$\frac{1}{8}$
	7th	7	$\frac{1}{7}$
	6th	6	$\frac{1}{6}$
	5th	5	$\frac{1}{5}$
	4th	4	$\frac{1}{4}$
	3rd	3	$\frac{1}{3}$
	2nd	2	$\frac{1}{2}$
	1st (Prime Tone)	1	1

[1] *A modulus is a constant factor by which we can change a thing from one system to another: if you want to express so many shillings in terms of pence, your modulus is* 12; *if in terms of sovereigns, your modulus is* $\frac{1}{20}$. *If a vibration-system is producing C and we wish it to produce e'', we must multiply its frequency by the factor* 10: *i.e.* 10 *is our modulus; if a pipe of a given length gives the note e' and we wish to know what length of pipe will produce c'', we measure the given pipe and multiply it by the modulus* $\frac{5}{8}$.

It will readily be seen that the use of vibration-fractions forces us to use multiplication and division in the addition and subtraction of intervals where, on the keyboard, musicians merely add or subtract. Thus a major third added to a minor third makes, as every musician knows, a perfect fifth. But if it is the vibration-fraction of a fifth that we want, we must *multiply* (and not add) the vibration-fraction of the major and minor thirds:

$$\tfrac{5}{4} \times \tfrac{6}{5} = \tfrac{6}{4} = \tfrac{3}{2}.$$

Example. Given the frequency of a note as 60, and the vibration-fraction of a fourth and major third as $\frac{4}{3}$ and $\frac{5}{4}$, what is the frequency of the major sixth from the given note?

Fourth + major third = major sixth.

$\frac{4}{3} \times \frac{5}{4}$ = vibration-fraction of major sixth $= \frac{5}{3}$.

Required frequency $= 60 \times \frac{5}{3} = 100$.

Again, if we have a minor sixth and wish to reduce it to a minor third, we *subtract* a perfect fourth; but in dealing with the vibration-fractions of the intervals we must *divide* instead of subtracting.

Example. Given C = 260 and A flat = 416, find the frequency of the E flat which lies between them, the vibration-fractions of a perfect fourth and minor sixth being $\frac{4}{3}$ and $\frac{8}{5}$.

$$\text{Vibration-fraction of minor third} = \tfrac{8}{5} \div \tfrac{4}{3} = \tfrac{8}{5} \times \tfrac{3}{4} = \tfrac{6}{5}$$

frequency of E flat $= 260 \times \frac{6}{5} = 312$.

We could have reached the same result in a shorter way by saying that since E flat is a fourth below A flat,

$$\text{frequency of E flat} = \text{frequency of A flat} \div \tfrac{4}{3} = 416 \times \tfrac{3}{4} = 312.$$

Fourier's Theorem establishes the fact that, however complex the wave-curve of a periodic vibration may be, it can always be resolved into a number of simple wave-curves selected from a series whose lengths form the harmonic progression 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, . . . &c.

The true application of curves to vibrations is dealt with later (c. 16 *seq.*), and at this point the student need only consider the meaning of Fourier's Theorem as applied to sounds:

Any musical sound which is not a simple tone *can* be resolved into a number of simple tones selected from a series whose vibration-numbers are in the proportion 1, 2, 3, 4, 5, . . . &c.

N.B. Fourier's Theorem does not imply that *all* of the series will be present. A compound tone may consist, say, of a prime tone modified by its sixth partial. But a compound sound must consist of *at least* two simple tones, and *may* include the whole series of partials.

Omit

CHAPTER XII

TIMBRE

THE fact that the sounds we hear are seldom simple tones, but are almost invariably composite sounds (or clangs) composed of a number of partial tones each in itself simple, suggests the question whether these partials are always present in exactly the same degree. When we hear middle C played first on a trumpet and then on a violin we know that the two sounds have certain things in common: their frequency is the same, their vibrations travel through the air at the same pace. But if we ask whether the two sounds are composed of the same partials, the answer is 'yes' as far as the *series* of partials belonging to middle C is concerned, 'no' in so far as each instrument makes its own selection and varies the relative intensity of those chosen. And it is precisely this variation in the number and intensity of the partials present in a given sound that accounts for differences of timbre or quality. We may state this fact as follows:

Two clangs of the same pitch must select their partials from the same series, but may differ in timbre:

(1) through selecting different partials—some of the series being absent in either or both sounds;

(2) through the different intensities of the various partials, should the selection happen to be identical;

(3) through variation both in selection and relative intensity.

The corollary to the above statement must obviously be that pure tones, having no partials, can only differ in pitch and intensity; and experiment proves this to be the case. A resonator, constructed to vibrate only when it is excited by the air-vibrations corresponding to its pitch, produces a simple tone, and it is impossible to distinguish between resonators in the matter of individual quality. Tuning-forks, again, produce tones which are very nearly pure, and they are almost exactly alike. The cooing of a dove, or the soft 'oo' sound of a well-trained choir-boy, are about as near pure tones as we can get in ordinary life.

A vibrating string generates all the partial tones; but the ear is only capable of recognizing—as in all clangs—a limited number of the lower ones. And since the vibrations of a string (especially of a large string vibrating with considerable amplitude) are easily visible to the naked eye, an examination of them forms a good introduction to the more intricate questions which arise later.

If you twitch the C string of a 'cello with some violence, it immediately vibrates, like a skipping-rope, between its two fixed ends; and these fixed ends, or points of rest, are called *Nodes*.

This vibration will, to the eye, take the shape of the double curve of fig. 23, where *a* and *b* are the nodes, the dotted line *ab* the position of rest of the string, and the dotted line *cd* the amplitude of the vibration. Before considering fig. 23 any further the student must be reminded of two facts:

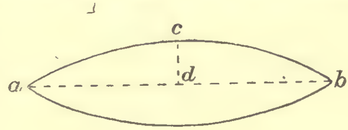


FIG. 23

(1) Although from a sideways view the string *looks* like fig. 23, nevertheless any point on the string is actually describing a circle. Thus the string will never touch or pass through its position of rest *ab* until the moment comes when it ceases to vibrate at all, and at that moment silence occurs.

(2) The movement we are considering is *only* the movement of the string. It must be kept quite distinct in the mind from any idea of the movement which takes place

in the air as a result of vibrations communicated to the air by the string.

The vibrations of a string, as illustrated in fig. 23, will result in a sound whose pitch is the prime tone of the note to which the string is tuned. That is to say, if the illustration represents the vibration of the C string of a 'cello, then the sound resulting would be C, a pure tone, and no partials would be present. But in reality the string, in addition to vibrating as in fig. 23, is vibrating in sections of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., at the same time, and the vibrations of these sections are resulting in sounds corresponding to the 2nd, 3rd, 4th, 5th, &c., partial tones of the note to which the string is tuned.

The whole conception of a string vibrating in all these innumerable sections, simultaneously yet independently, is at first so difficult that many students abandon all attempts at

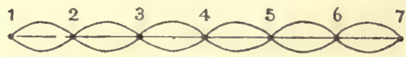


FIG. 24

comprehending it. Possibly the following explanation may make the process a little less bewildering.

Let us suppose that seven people take up their positions in a straight line at equal distances from one another, and then begin to turn, at the same instant and same pace, six exactly equal skipping ropes. A front view will give us a picture of six exactly equal double curves (fig. 24). Instead of calling these seven agents 'people', let us call them 'nodes', and by taking away the earth imagine the process to be going on in space.

Suddenly four nodes—*a*, *b*, *c*, *d*—of a superior species appear and, taking the position of nodes 1, 3, 5, 7, begin, *in addition* to the movement already going on, to turn the two-rope sections between each of them as a single skipping-rope, the pace of such new turning being half as fast as the original pace.

The result, at the moment when each swing is at its highest point, can be imagined from fig. 25.

At this juncture three nodes of a still higher species arrive, and take their places at the two ends and the middle point (occupied in fig. 25 by *a*, *d*, and 4). They then swing the ropes in two sections at one-third the original pace, each comprising three of the original rope-lengths, and the resultant curve of these two similar sections cannot be suggested pictorially, though mathematicians could express it as an equation.

Finally two giant nodes, seizing the two ends, swing the rope as a whole, at one-sixth the original pace.

If we could get a front view of the above performance we should see a rope vibrating, at the same time, (1) as a whole, (2) in half sections, (3) in one-third sections, (4) in one-sixth sections; and the result, if the swinging were as rapid as the

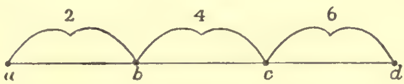


FIG. 25

swing of the C string of the 'cello, would look, in spite of the four separate systems at work, like fig. 23 with a blurred outline.

Whether or no the above explanation helps the student to realize the motion of a string, the fact must be grasped that every string does vibrate

(1) as a whole, with nodes at each extremity; producing the prime tone of the string;

(2) in halves, with a further node at the centre of the string; producing the 2nd partial;

(3) in thirds, adding two nodes which divide the string into three equal parts; producing the 3rd partial;

&c., &c.

It may be remarked here that normally the partials decrease in intensity as they rise in pitch—i.e. as their distance from the prime tone increases; and though in some special cases quality of tone is due to the prominence of certain higher partials, yet we can say that very high partials are practically negligible. Consequently, in vibrating strings, the amplitude of swing of the overtones soon becomes so small that its influence on the vibration as a whole may be ignored.

The vibrations of pipes are vibrations in the air itself. An organ flue-pipe does not, like a string, *cause* the air-vibrations, but only

(1) controls the rapidity, by the length of the column of air within it;

(2) modifies the tone of the resultant sound, by its shape and structure.

Consequently the full understanding of the action of the air must be delayed until the Transmission of Vibrations is dealt with. But the actual influence of the pipe on the vibrations, especially in the matters of wave-lengths and partials, can be brought out by the analogy of the skipping-rope and its nodes, *provided always* that the student grasps the fact that the curves drawn in the illustrations do not actually occur in the pipes at all.

In dealing with pipes a double curve is used to represent one complete vibration in the air.

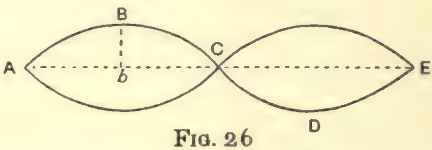


FIG. 26

In fig. 26 ACE represents the length of one vibration; the air being compressed in the half ABC, rarefied in the half CDE, and normal at the points A, C, and E, which are therefore nodes.

Open pipes. In open pipes—i. e. pipes open at both ends—the one essential fact to be remembered is that the ends *cannot* be nodes (i. e. places where the air is in a normal state) but *must* be antinodes (i. e. places where the air is in a state of maximum compression or rarefaction). The nodes in a skipping-rope are the two ends, the antinode is the middle point where the amplitude of the swing is greatest.

When, therefore, the air in an open pipe is vibrating—as it always *can*—with only one node, that node will be in the middle of the pipe (fig. 27).

FIG. 27

In this case the wave-length of the vibrations will be the greatest that the pipe is capable of producing; and since it is clear from fig. 27 that only half a vibration (see fig. 26) is created, it follows that the wave-length is twice the length of the pipe. And the note produced, since the pipe cannot make the wave-length any greater, is the prime tone of the pipe.

But an open pipe can also contain an air-wave with two, or three, or any number of nodes, provided only that the one essential condition—that the two ends are antinodes—is fulfilled. Figs. 28 and 29 show a pipe with two and three nodes respectively:

FIG. 28

FIG. 29

The vibrations illustrated in the above example will clearly have wave-lengths a half and a third as long, respectively, as that in fig. 27; and these are the exact wave-lengths required for the second and third partials of the prime tone.

Hence an open pipe can produce all the partial tones of the Harmonic Chord.

Stopped Pipes. A stopped pipe is one which is stopped at one end, the other end (where the 'lip' is) being open.

When the column of air in the pipe is in a state of vibration the same law holds as in the case of open pipes: that the open end of the pipe cannot be a node, but must be an antinode. The stopped end must be a node, and cannot be an antinode.

Figs. 30, 31, and 32 illustrate the vibration in three stopped pipes of equal dimensions when one node (fig. 30), two nodes (fig. 31), and three nodes (fig. 32) have formed themselves.

It will be seen that in fig. 30 the actual length of the air-vibrations will be four times as long as the pipe, since the curve in the pipe only represents one-quarter of the full curve representing a whole vibration (see fig. 26). Thus the fundamental note, or prime tone, of a stopped pipe is the sound whose wave-length is four times the length of the pipe, and this is the lowest note the pipe can produce, since the air in it *must* have at least *one* node and *one* antinode.

FIG. 30

FIG. 31

FIG. 32

When there are two nodes and two antinodes, as in fig. 31, the next lowest note (i.e. the next longest wave-length) will be produced; and this is represented by the curve which is three-quarters of the whole vibration-curve of fig. 26. This clearly represents a sound whose wave-length is one-third of the fundamental; since the wave-length of fig. 31, when completed, will be one-third the completed wave-length of fig. 30. And this means that the vibration-number of the note being produced in fig. 31 is three times as great as that of the note being produced in fig. 30: in other words, that the note of fig. 31 is the third partial of the prime tone.

Similarly when, as in fig. 32, we introduce three nodes and three antinodes, the resulting note will be one-fifth the wave-length of the prime tone, will have five times the frequency, and will be the fifth partial.

From the above description the student should grasp the important fact that whereas the air in an open pipe can vibrate in segments in such a way that *all* the partials are present in the resulting sound, a stopped pipe can only produce a sound containing partials 1, 3, 5, 7, 9 . . . .

Further, it should be noticed, as bearing on the character of the sounds produced by stopped pipes, that in the series of partials of the complete Harmonic Chord—

(1) Every odd partial introduces a new note to the chord;

(2) No even partial introduces a new note, each being merely the octave above some other partial.

We can now obtain the note of an open or stopped pipe when once we know the exact length of it.

Supposing an open pipe is 8 feet long, we know the wave-length of its prime tone will be 16 feet. Consequently, if we take the velocity of sound as 1,100 feet per second, the number of vibrations of this note will be

$$\frac{1100}{16} = \frac{275}{4} = 68\tfrac{3}{4}.$$

This figure is, as we should expect, very close to the lowest C on the manuals of the organ—a note which is often called '8 foot C'.

Had the pipe been stopped the wave-length would have been 32, and the vibration-number $34\tfrac{3}{8}$, giving the note C, i.e. practically the bottom note of an ordinary pedal-Bourdon.

Written as formulae, for conciseness, we get

(1) for open pipes: $n^* = \dfrac{1100}{2l}$;

(2) for stopped pipes: $n = \dfrac{1100}{4l}$.

Every organist is aware of the fact that a stopped pipe gives a note an octave lower than an open pipe of the same length, but they are often not clear about the reason; many of them, indeed, imagining vaguely that the sound travels

* n = number of vibrations.

along the pipe and, finding its exit barred, has to make a double-journey by coming back. But this is a thoroughly unscientific misconception; the real—and quite simple—fact being that in an open pipe the node of the prime tone *must* be in the middle, in a stopped pipe it *must* be at the stopped end of the pipe.

The sounds that can be produced from rods, plates, bells, &c., are of interest and importance to physicists rather than to musicians; and if the reader desires to inquire into them he will find details in any good book on Sound written from the scientific side. If, however, he should ever find himself in a physical laboratory where there is any acoustical apparatus, he is specially advised to make some elementary experiments with Chladni's plates. These consist of thin pieces of metal or glass, of square or circular shape, with smooth surface. They are fixed into a vice by means of a short rod welded to them at their centre. When a violin-bow 'plays' on the edge they vibrate and give quite a musical sound; and if a fine sand is sprinkled on the surface it will, when the vibration begins, scamper all over the plate and finally settle down, in the most beautiful and symmetrical patterns, along nodal lines where the surface is at rest.

The **Flute** is an ordinary open flue-pipe. The mouth forces a column of air against the 'lip' of the flute, producing a tangle of vibrations. The flute then chooses, according to its length or the system of nodes arranged by the fingers, the particular vibration-rate with which it is in sympathy, and reinforces it.

All the partials are possible, but it is practically only the prime tone and second partial that can be detected.

The **Clarinet** is a pipe of uniform bore, whose air-column or node-system (as arranged by the fingers) governs the rate of the vibrations of a beating reed. It is not properly a stopped pipe, but it is generally stated that it produces a sound comprising the odd partials only, and so must be considered as acting like a stopped pipe. The second partial is certainly absent, but as Helmholtz found distinct traces of Nos. 6 and 8, the pipe may be considered as acting in an irregular manner.

The **Oboe** and **Bassoon** are open pipes of conical bore, and their sounds comprise the whole of the ordinary partials, the intensity of which decreases fairly regularly as the series ascends.

The usual **Brass Instruments,** played by means of cup-shaped mouthpieces in which the lips act as reeds, produce sounds containing all the partials; and by manipulating the lips we can induce the tube to reinforce any one of the overtones (within limits) instead of the fundamental note. In many of them, indeed—such as the Bugle—it is quite easy to produce an overtone, and exceedingly difficult to sound the fundamental note at all. Pistons, by increasing the length of the air-column, lower the pitch of the prime tone, and so provide us with an entirely new set of available overtones.

May 16

PART V. TEMPERAMENT

CHAPTER XIII

ON THE TWELFTH ROOT OF TWO

WHEN you multiply a number by itself you are said to 'square' it, or to raise it to the power of two; and if you multiply the result by the original number, then the original number is 'cubed', or raised to the power of three.

Thus 9 may be written as 3^2, which is read 'Three squared' or 'Three to the power of two'; and 8 may be written 2^3, i.e. 'Two cubed' or 'Two to the power of three'.

The number which tells us to what power the principal number is to be raised is called the *Index* (plural *Indices*): so that in the expressions 2^8 and 3^9, the indices are 8 and 9. Almost every one has a working knowledge of the meaning of indices, so long as they are whole numbers, but when they are fractions the non-mathematical mind is apt to be baffled. Fractional indices, however, really stand for something quite easy to grasp, and it is essential that any one who aims at a full understanding of Temperament should not be puzzled by the elementary facts about them.

These facts fall into three groups:

(1) If we take any number with any index—for simplicity let us say 3^2—and multiply it by itself ($3^2 \times 3^2$), the answer is found by *adding* the indices—$3^{2+2} = 3^4$.

This is also true when the indices are different, so long as the principal number is the same:

$$2^2 \times 2^3 = 2^{2+3} = 2^5.$$

That this fact must always hold good will probably seem evident to any one who asks himself exactly what is meant by $2^2 \times 2^3$. For

$$2^2 = 2 \times 2.$$
$$2^3 = 2 \times 2 \times 2.$$

Consequently $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{2+3}$.

The algebraical way of expressing the above fact, which is always true, is by saying that

$$a^x \times a^y = a^{x+y}.$$

(2) If you wish to take the square root of a number you can show it by using the index $\frac{1}{2}$. Thus the square root of 9 can be written as $9^{\frac{1}{2}}$.

This ought to be clear to any one who realizes that 9^1 (nine to the power of one) is 9. For

$$9 = 9^1 = 9^{\frac{1}{2}+\frac{1}{2}} = 9^{\frac{1}{2}} \times 9^{\frac{1}{2}}.$$

And if $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$ it is obvious that $9^{\frac{1}{2}}$ must be the square root of 9.

Similarly, the cube root of 2 can be written $2^{\frac{1}{3}}$, the fourth root of x^3 becomes $x^{\frac{3}{4}}$, the twelfth root[1] of two, $2^{\frac{1}{12}}$.

(3) The same rule applies to the multiplication of numbers with fractional indices as to those with whole numbers. Thus, if the square root of 4 is to be multiplied by itself:

$$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{\frac{1}{2}+\frac{1}{2}} = 4^1 = 4.$$

Again, if the cube root of 8 is to be multiplied by itself:

$$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{2}{3}} \text{ (i.e. cube root of } 8^2\text{, which is 4).}$$

Similarly, if we multiply the twelfth root of 2 by itself:

$$2^{\frac{1}{12}} \times 2^{\frac{1}{12}} = 2^{\frac{1}{12}+\frac{1}{12}} = 2^{\frac{2}{12}} = 2^{\frac{1}{6}}.$$

The student should verify for himself the results in the following table:

[1] The twelfth root of 2 is also written $\sqrt[12]{2}$.

$2^{\frac{1}{12}}$ raised to the power of		$2 = 2^{\frac{1}{6}}$
,,	,,	$3 = 2^{\frac{1}{4}}$
,,	,,	$4 = 2^{\frac{1}{3}}$
,,	,,	$5 = 2^{\frac{5}{12}}$
,,	,,	$6 = 2^{\frac{1}{2}}$
,,	,,	$7 = 2^{\frac{7}{12}}$
,,	,,	$8 = 2^{\frac{2}{3}}$
,,	,,	$9 = 2^{\frac{3}{4}}$
,,	,,	$10 = 2^{\frac{5}{6}}$
,,	,,	$11 = 2^{\frac{11}{12}}$
,,	,,	$12 = 2$

If the elementary theory of indices is understood up to this point no student should find it difficult to realize the truth of the following statement (on which the whole structure of equal temperament is based):

When the number 1 has been twelve times multiplied by the twelfth root of 2, the result is 2; when the number 2 has been so multiplied twelve times it becomes 4; the number 4 so multiplied becomes 8; and the numbers reached after each process of twelve multiplications form the Geometrical Progression 1, 2, 4, 8, 16, &c., &c.

For the sake of simplicity in working a method has been devised of measuring intervals by means of *Cents*, there being 1,200 cents in an octave and 100 in a semitone (in equal temperament).

The understanding of cents depends on the recognition of one outstanding fact in the table at the head of this page. In the column on the left of the signs of equality the terms increase by means of *multiplication*; whereas in the column to the right the terms, though increasing by exactly the same amounts, show a series of indices which are in Arithmetical

Progression ($\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, &c.)—i. e. the increase is represented by *addition*.

The greater simplicity, in dealing with intervals, of being able to use addition instead of the multiplication and division involved by the vibration-ratios shown on p. 70, has led to this system of cents; but the student is warned that the mathematician, in saying there are 100 cents in a semitone, means something different from what he *appears* to be saying. That is to say, he does not mean that the distance between C and C sharp can be called '100 cents', and the distance between G and G sharp called the same; for he knows quite well the two distances are different. But he means that, since we can get C sharp and G sharp from C and G by using the same multiplier in each case, we can, *by thinking of indices and not multipliers*, use addition instead of multiplication.

[Readers who have any elementary knowledge of logarithms will have guessed how this substitution is effected. A convenient unit was chosen for the cent, such that 1,200 cents corresponds to log 2. If we then wished to find the number of cents in a semitone of equal temperament we can say:

$$\begin{aligned}\log(1 \times 2^{\frac{1}{12}}) &= \log 1 + \log 2^{\frac{1}{12}}\\ &= 0 + \tfrac{1}{12}\log 2\\ &= \tfrac{1}{12}\,(1200 \text{ cents})\\ &= 100 \text{ cents.}\end{aligned}$$

Similarly for the next semitone:

$$\begin{aligned}\log(2^{\frac{1}{12}} \times 2^{\frac{1}{12}}) &= \log 2^{\frac{1}{6}}\\ &= \tfrac{1}{6}\log 2\\ &= 200 \text{ cents.}\end{aligned}$$

A fuller explanation is afforded by the following extract from Professor Barton's *Text-book on Sound*:[1]

'Since pitch depends upon frequency and interval upon ratio of frequencies, we have the following important result. Let it be

[1] Printed by kind permission of the author.

required to measure intervals so that the sum of the measures of component intervals shall be the measure of the resultant interval. Then the only measure possible is that of taking for each interval a number proportional to the logarithm of the ratio of the frequencies of the notes composing that interval. Thus, let the frequencies of three notes, beginning at the highest and proceeding in order of pitch, be L, M, and N. Also let the intervals be I_1 between L and M, I_2 between M and N, and I between L and N. Then, if each interval be measured by k times the logarithm of the ratio of frequencies, we have

$$I_1 = k \log \frac{L}{M} = k (\log L - \log M) \quad . \quad . \quad . \quad . \quad (1)$$

$$I_2 = k \log \frac{M}{N} = k (\log M - \log N) \quad . \quad . \quad . \quad . \quad (2)$$

$$I = k \log \frac{L}{N} = k (\log L - \log N) \quad . \quad . \quad . \quad . \quad (3)$$

But by addition of (1) and (2)

$$I_1 + I_2 = k (\log L - \log N) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so by (3) and (4)

$$I = I_1 + I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For k any convenient number could be chosen, as (5) shows that the relation desired is independent of it. But the late Mr. A. J. Ellis (the translator of Helmholtz's *Sensations of Tone*) has adopted as the unit for this logarithmic measure the *cent*, 1,200 of which make the octave. The name cent is used because 100 cents make the semitone of those instruments in which twelve equal semitones are the intervals occurring in an octave. Hence the clue to reduction of any intervals to these logarithmic cents would be found in the following equations, where I is the interval in cents between notes of frequencies M and N:

$$I = k \log \frac{M}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$1200 = k \log 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Whence by (6) ÷ (7)

$$I = 1200 \, \frac{\log M - \log N}{\log 2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)\text{']}$$

CHAPTER XIV

EQUAL AND MEAN-TONE TEMPERAMENT

It is quite easy to find, or to write, a short passage of music in a major key without any accidentals. Such a passage is called 'diatonic', and can be played on the piano or organ in the key of C—i. e. on the white notes alone.

Now if we once fix on the pitch of one note, we can, by using the table of vibration-fractions on p. 70, tune every white note on a keyed instrument in perfect accordance with the rules there systematized; and such an instrument is said to have 'just', 'true', or 'perfect' intonation. Let us fix on middle C (c') as a note whose frequency is 264. Then the vibration-numbers of the eight notes forming the major scale upwards from middle C will be as follows:

Tonic	528	that is 264×2
Leading note	495	,, ,, $\times \frac{15}{8}$
Superdominant	440	,, ,, $\times \frac{5}{3}$
Dominant	396	,, ,, $\times \frac{3}{2}$
Subdominant	352	,, ,, $\times \frac{4}{3}$
Mediant	330	,, ,, $\times \frac{5}{4}$
Supertonic	297	,, ,, $\times \frac{9}{8}$
Tonic	264	

A diatonic passage played on an instrument tuned in this way produces an effect which is a revelation of smoothness to any one hearing it for the first time. The organ mentioned on p. 71 produces one chord whose intonation is just; and is also fitted with pipes which will give the same chord tuned in the usual way. If, after listening for some time to the pure chord, the tempered chord is sounded, it is so manifestly out of tune that the listener can only wonder that human beings can be found to tolerate it.

But a very little simple arithmetic is needed to convince us that the tuning system suggested above is untenable, on a keyed instrument, when we begin modulating, however beautiful it may be when used for diatonic music. And its unworkableness can be seen from an examination of three intervals: the major second, major third, and perfect fifth.

Major second. Let us construct the first three notes of a major scale whose tonic has a frequency of 64 (which is the 'philosophic pitch' of C). The notes will be

$$E = 80 \text{ (i.e. } 64 \times \tfrac{5}{4})$$
$$D = 72 \text{ (i.e. } 64 \times \tfrac{9}{8})$$
$$C = 64.$$

If at any time we wish to modulate to the scale of D (and D minor would be a very natural modulation) we shall want a scale whose tonic and supertonic are

$$E = 81 \text{ (i.e. } 72 \times \tfrac{9}{8})$$
$$D = 72.$$

Thus the E which we have tuned for the mediant of C will not be in tune as the supertonic of D; and though the difference of one vibration looks small on paper it would be very noticeable in practice, for F has a frequency of $85\frac{1}{3}$, and the difference between 80 and 81 is far from negligible when we compare these numbers with $85\frac{1}{3}$, however small it may seem in itself.

In a *true* major scale, then, the interval from tonic to supertonic, one whole tone, is greater than the interval from supertonic to mediant, though the latter is also called a whole tone; and the former is called a *major tone*, the latter a *minor tone*. They differ in the ratio 81 : 80; and this difference, expressed as a fraction, $\frac{81}{80}$, is called a *Comma*—sometimes known as the Comma of Didymus.

Major third. If we start with $C = 64$ we know that E will have for its frequency $64 \times \frac{5}{4} = 80$.

From E = 80 we can say with certainty that a true G sharp has a frequency of $80 \times \frac{5}{4} = 100$.

Similarly from G sharp we get B sharp $= 100 \times \frac{5}{4} = 125$.

Now if C = 64 the octave higher *must* have a frequency of 128, yet we find that three major thirds from C give us a note B sharp with a frequency of 125. These two notes differ in the ratio 128 : 125; and this difference, expressed as a fraction $\frac{128}{125}$, is called an *Enharmonic Diesis*.

Perfect Fifth. If we take a note with n vibrations we know that the note one octave higher has $2n$ vibrations, the octave above that $4n$, and so on; the various octaves having vibration-numbers which form the geometrical progression n, $2n$, $4n$, $8n$, $16n$ . . .

When we reach the note seven octaves higher than our original note, we shall find its frequency to be $128n$.

Let us suppose the lowest C on the piano to have n vibrations, and, consequently, the highest C (the top note on the piano) to have a frequency of $128n$.

Now it is evident, on the piano, that we can reach from the lowest C to the highest, not only by seven jumps of an octave (multiplying the frequency by 2 at each jump), but also by twelve jumps of a perfect fifth—multiplying the frequency by $\frac{3}{2}$ at each jump. In this way the vibration-number of the top C will prove to be

$$n \times (\tfrac{3}{2})^{12}.$$

Any one who will trouble to work out this sum will find that it gives an answer of nearly $130n$: so that the two notes reached by pure tuning are not, as they are on the piano, identical.

The difference between the two notes, one of them (the higher) twelve intervals of a fifth from a given note, the other seven intervals of an octave, is expressed as a vibration-ratio: $(\frac{3}{2})^{12} : 2^7$, which, expressed as a fraction, is (about) $\frac{74}{73}$, and this fraction is known as the *Comma of Pythagoras*.

If we were to try—and there have been many attempts—to construct a keyed instrument on which we could play in perfect tune in any key, we should find, by the most modest calculations, that every octave would have to contain at least eighty different notes. Instruments with adjustable pitch—such as the voice or violin—can at once provide us with any of our eighty notes, the ear of the performer slightly sharpening or flattening a note whenever a new tonality asserts itself.

For instance, a really good choir, accustomed to singing without accompaniment, would sing such a passage as the following (fig. 33) with just intonation:

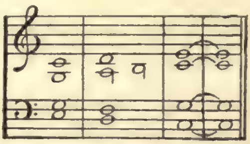


FIG. 33

But should the harmony change, in the last bar, so as to suggest the tonality of D minor (as in fig. 34), then the sopranos would beyond question alter the pitch of the note E, readjusting it so that the note which they began as the mediant of C they will leave as the supertonic of D:

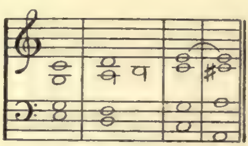


FIG. 34

On keyed instruments, however, the difficulties of construction and execution make it impossible even to consider such an enormous number of notes to the octave; and the history of the keyboard is largely a history of the attempts of musicians to construct a feasible system by making compromises with the intractable laws of nature. Such compromises, of course, necessarily took the form of tampering with true intonation, generally by 'splitting the difference' between two notes whose frequencies differed by very little, and then making the one sound do the work of both notes.

Any interval thus deliberately tuned contrary to the exact vibration-fractions of just intonation is a '*tempered*' interval; and any system of tuning which aims, by the use of tempered intervals, at reducing the number of necessary notes in the octave, is called a '**Temperament**'.

Two such systems, **Mean-tone Temperament** and **Equal Temperament,** surpass all others in historical and practical importance, and the general principles on which they rest will now be described.

Mean-tone Temperament. Up to early mediaeval times the chief system in vogue was the Pythagorean, which sacrificed the accuracy of the major third in order to preserve that of the perfect fifth. It required twenty-seven notes to the octave. Mean-tone Temperament was devised in order that instruments with fixed notes might be tuned on a more practical system.

It must be remembered that mediaeval music was modal, and that composers did not require a large number of key-notes, since modulation and transposition were both kept within very small limits. Consequently it was thought better to preserve pure intonation, as far as possible, in the keys most commonly used, thereby sacrificing it in the unusual keys, even to the extent of making it impossible to use such keys at all.

The essential fact of this system (from which fact the system derives its name) is that the difference between the major tone and minor tone (see p. 92) is abolished. This means that the true major third was retained, and that the two whole tones comprising it were equidistant.

The above fact, expressed mathematically, comes to this. If a note chosen as tonic has a vibration-number n, its mediant will have a frequency $(n \times \frac{5}{4})$. The supertonic, which in just intonation has a frequency $(n \times \frac{9}{8})$, is given instead a frequency of $\left(n \times \frac{\sqrt{5}}{2}\right)$. The advantage lies in this: that

$$\text{one whole tone from } n \qquad = n \times \frac{\sqrt{5}}{2}$$

$$\text{one whole tone from } n \times \frac{\sqrt{5}}{2} = n \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}$$

$$= n \times \left(\frac{\sqrt{5}}{2}\right)^2$$

$$= n \times \frac{5}{4}$$

and $n \times \frac{5}{4}$ is, as we postulated, the true mediant, and that little discrepancy called the comma of Didymus $(\frac{81}{80})$ never shows its head at all.

The frequencies of the notes of the major scale, in this system, were as follows:

Tonic	$2n$.
Leading note	Dominant frequency $\times \frac{5}{4}$.
Superdom.	Subdominant frequency $\times \frac{5}{4}$.
Dominant	$n \times \frac{3}{2}$ flattened a quarter of a comma.
Subdom.	$2n \times \frac{2}{3}$ sharpened a quarter of a comma.
Mediant	$n \times \frac{5}{4}$.
Supertonic	$n \times \frac{\sqrt{5}}{2}$.
Tonic	n.

The above table will be better understood when it is explained how, in this system, the perfect fifth is found.

Remembering that the inexorable condition of the system is that the major thirds are true, we can find from any note the exact pitch of its fifth partial. In fig. 35, for instance, we know that the frequency of e'', assuming philosophic pitch, will be $128 \times 5 = 640$.

But this interval (c to e'') is exactly four perfect fifths, and if we use the vibration-fraction of a fifth the note e'' will be

$$128 \times (\tfrac{3}{2})^4 = 128 \times \tfrac{81}{16} = \tfrac{128}{16} \times 81,$$

whereas $$640 = 128 \times \tfrac{80}{16} = \tfrac{128}{16} \times 80.$$

Fig. 35

Thus it appears that the difference between the two sounds that we get for e'' is a difference between 81 times something and 80 times the same thing: i. e. the difference is one comma. And so, as four perfect fifths give us a note one comma sharp, each fifth, in Mean-tone Temperament, is flattened by exactly a quarter of a comma.

We could, consequently, tune the scale of C, Mean-tone Temperament, by the vibration-fractions of (1) a tempered whole tone $\left(\frac{\sqrt{5}}{2}\right)$, (2) a true major third ($\frac{5}{4}$), (3) a tempered fifth (one quarter-comma flat).

C . . 128.
B . . G frequency × $\frac{5}{4}$.
A . . G frequency × $\frac{\sqrt{5}}{2}$.
G . . 64 × $\frac{3}{2}$ (flattened one quarter-comma).
F . . 128 × $\frac{2}{3}$—i. e. a fifth below 128—(sharpened one quarter-comma).
E . . 64 × $\frac{5}{4}$.
D . . 64 × $\frac{\sqrt{5}}{2}$.
C . . say 64.

To the above are added:

F sharp, a true major third above D.			
C sharp,	,,	,,	,, A.
G sharp,	,,	,,	,, E.
B flat,	,,	,,	below D.
E flat,	,,	,,	,, G.

This completes the 12-note scale, and gives us six keys in which, except for slightly unsatisfactory fifths, we can play with ease and comfort. Indeed, the error of a quarter-comma in the fifths is really negligible, as it is doubtful whether any human ear could possibly distinguish it; and so in the six keys provided by the 12-note octave of Mean-tone Temperament we can say that we are playing with just intonation.

The drawback to the above system lies in this: that although music in the keys provided for will sound exceptionally beautiful, any attempt to go beyond those keys—say to E major or A flat major—is almost impossible. The notes D sharp and A flat do not exist, and in place of them we should be compelled to use E flat and G sharp, and both the latter notes will be so badly out of tune that the effect of the chord containing such a substitute is known as a 'wolf'. The discrepancy between G sharp and A flat is an enharmonic diesis, for

$$\begin{aligned}\text{frequency of G sharp} &= \text{frequency of C} \times \tfrac{5}{4} \times \tfrac{5}{4},\\ &= \quad ,, \qquad ,, \quad \times \tfrac{25}{16},\\ &= \quad ,, \qquad ,, \quad \times \tfrac{125}{80};\\ \text{frequency of A flat} &= \text{frequency of C} \times \tfrac{4}{5} \text{ (to get major third } \textit{below}\text{)},\\ &= \quad ,, \qquad ,, \quad \tfrac{64}{80},\end{aligned}$$

and $\frac{64}{80}$, raised an octave, becomes $\frac{128}{80}$.

Thus the vibration-fraction of the interval between A flat and G sharp is $\frac{128}{125}$, a very noticeable difference; and that between D sharp and E flat is even more considerable, the E flat being nearly two-fifths of a semitone sharp.

Old organs used to be provided with two extra black-keys for D sharp and A flat, and such extra keys are still to be found on the concertina, the only instrument in which Mean-tone Temperament survives; but even then the remoter keys were impossible, and many chords which, though they involve accidentals, might nowadays almost be called diatonic were barred, even in the more usual keys for which the instrument was supposed to be adequate.

If we look on an organ as an instrument for the accompaniment of Church music written in reasonably diatonic style, then it is undoubtedly a pity that the extraordinary sweetness of the Mean-tone Temperament has been sacrificed. But if we consider the organ as a solo instrument, or claim that Church music should gradually absorb the complexities of modern music, then we must admit that a keyboard which requires twenty-one notes to the octave is an impossible anachronism.

Equal Temperament. The world was gradually, though reluctantly, forced to the conclusion that it was necessary to have a keyboard on which it was possible to play music in any key, and that for this purpose an octave of twelve equidistant semitones was the only feasible solution. And one of the many services of J. S. Bach to music lies in the fact that the whole weight of his influence was thrown into the scale on behalf of Equal Temperament, and that the composition of the 48 stands out in history as the death-blow to all other systems of tuning.

The essential point in Equal Temperament being that the gap between any note and its octave is filled by eleven equidistant notes, it may be well to point out at once the one and only difficulty in grasping the system.

If we are asked to insert, between 12 and 24, eleven other numbers in such a way that the whole series of thirteen are equidistant, it is obvious that we merely have to count, like a child, 12, 13, 14, 15 . . . . 21, 22, 23, 24.

Similarly, if we have two pipes 12 inches and 24 inches in length, and have to make a series of thirteen with the same difference between each pipe-length, we should make their lengths 12, 13, 14 . . . . 22, 23, 24 inches.

But if we are given two organ-pipes 12 inches and 24 inches in length, and are asked to insert between them eleven pipes whose *notes* will be equidistant, in the sense that the intervals between them will be the same, we are faced by an entirely different problem.

Now many people fail to see, at first, why a constant distance, which satisfies the former experiment, will not satisfy the latter case. The reason, which is very simple, is as follows.

The notes produced by the two pipes of 12 and 24 inches will be an octave apart. If we could get our eleven semitones by means of pipes of 12, 13, 14, 15 . . . . inches, then the tops of the four largest pipes would appear, as in fig. 36, with a constant difference:

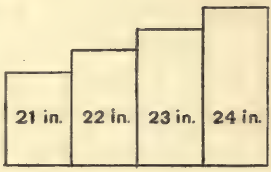


FIG. 36

Similarly, two pipes of 24 and 48 inches would give two notes an octave apart, and if we could insert our eleven semitones by making a series of equal additions to the length, the pipes would be 24, 26, 28, 30 . . . 42, 44, 46, 48 inches long. The tops of the first four would look like fig. 36, only the difference between successive pipes would be twice as much.

Now clearly we could, by placing the two lots of pipes together, get a series of semitones covering two octaves, and the tops of the five centre pipes would appear as in fig. 37:

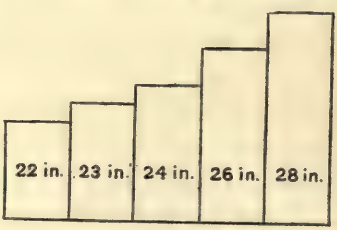


FIG. 37

A glance at this illustration will show that, while an increase of an inch per pipe has given us a difference of a semitone in pitch for a whole octave, we suddenly have to double our difference to two inches, and in the third octave will suddenly have to double it again.

The same argument would apply to vibration-numbers. If we had two notes whose frequencies were $12n$ and $24n$ their sounds would be an octave apart. Were it true that we could get a series of equidistant semitones by the notes whose frequencies are $12n$, $13n$, $14n$, $15n$ . . . . then to get a semitone *above* $24n$ we should suddenly be obliged to have a note $26n$, because the next octave will take us to $48n$.

The truth is, of course, that we have been trying to do with an arithmetical what can only be done by a geometrical progression.

We are given three terms in a series, the 1st, 13th, and 25th, and are required to fill in the others.

Were these three given terms 12, 24, and 36, we should be quite right in locating it as an arithmetical progression and merely filling in the numbers as a child would count them.

But since the three given terms are 12, 24, 48, the series *must* be in geometrical progression, and the difference between each term is not caused by addition, but by multiplication: that is to say, there is not a constant difference between successive terms, but a constant ratio.

The problem of Equal Temperament in its simplest possible form is this: if a note has a frequency of 1 its octave will have a frequency of 2. How can we insert eleven terms in geometrical progression between 1 and 2?

The series will be as follows:

$$1, f, f^2, f^3, f^4, f^5, f^6, f^7, f^8, f^9, f^{10}, f^{11}, f^{12}.$$

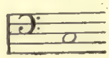
Now we know by hypothesis that the thirteenth term is 2. Therefore

$$f^{12} = 2,$$
$$f = \sqrt[12]{2},$$

and in Equal Temperament, if n represents the vibration-number of a note, then the vibration-number of the semitone above it is

$$n \times 2^{\frac{1}{12}}.$$

The musician will be able to think of many problems which arise from the fact that in Equal Temperament the major third is noticeably sharpened, whilst the perfect fifth is—though almost imperceptibly—flattened.

When, for example, the note C is struck on the piano, a trained ear will notice that the fifth partial stands out very clearly. If the note *c*  is held down on the organ, before long the note *e''*  will ring out, at the pitch of just intonation, with such insistence that it cannot be ignored. Should the note *e* then be added to the *c* already held down the shock momentarily caused by its false intonation is very unpleasant. Again, if the note *e* is held down the trained ear will, after a few moments, hear *b'* and *g''* sharp with the utmost clearness. Should the note *c* then be added below it, it will (both itself and its third partial *g'*) so 'curdle' with the partial-series of *e* that the immediate impression is one of intense discord.

The conclusion to be drawn is, that only by deliberately corrupting our ears from childhood onwards do we ever come to look on the opening major third of the Dead March—when played on a keyed instrument—as being in any way a concord. Luckily it seems to be one of the properties of buildings 'good for sound' that they select from the multitudinous vibrations of a chord those which are pleasant, and reject or swamp those 'curdlings' between more or less subsidiary overtones before they have had time to establish definite unpleasantness. Otherwise the numerous 'wolfs' due to the clash between prime sounds equally tempered and overtones in perfect tune would make even the ordinary major chord on a keyed instrument insupportably offensive.

One practical example of the realization of the dangers mentioned above will be of interest to organists. Organ-builders are in the habit of tuning mixtures by perfect intonation, and consequently have in general been wise enough to avoid introducing the major third—i.e. the 5th, 10th, and 20th partial. The fifth from the prime (i.e. the 3rd, 6th, 12th, and 24th partial), being almost imperceptibly out of tune with the tempered fifth, is used freely.

At this point one other matter of perpetual interest to musicians may be discussed—the question of the existence of key-colour, that individual character distinguishing one key from another which every musician claims to feel, and few will admit to be an illusion.

The writer, after inquiries long enough and wide enough to be called fairly exhaustive, feels justified in claiming that the differences in key-colour are attributable, not to any actual difference between keys, but to two groups of facts, one mechanical and the other psychological.

(1) **Mechanical.** A diatonic passage played on the piano in the key of C and then played in the key of D flat creates two different impressions on a listener. The sceptic says at once that the difference is due to change of pitch alone; but his claim is upset by the undoubted fact that when two pianos are at hand, the C of one tuned exactly to the D flat of the other (so that the passage can be played in the two keys at the same pitch), a trained musician can almost always say with certainty which performance employed only the white notes.

The writer has never come across any one capable of making the above distinction when the organ takes the place of the piano. If the pitch of an organ is a semitone sharp (as is still often the case) a listener has never (within the writer's experience) been known to say, 'That piece was in D flat major, but the performer was playing in C'. And this is exactly what, in the case of the piano, many listeners can say with certainty.

There seems to be one explanation of the above fact, and one only. The black keys of the piano are smaller and more awkwardly situated than the white ones; and, in addition, their length from the hammer is less. Hence the firmness and strength of stroke on the part of the player are affected, and also the action of the hammer, by the reduction of leverage, is modified. And these considerations do not in any way affect the sound of the organ-pipe.

One example should make this clear. The two chords of fig. 38 contain six notes:

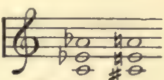


FIG. 38

If a machine were devised to register the exact strength of the six hammer-blows when these chords are played on a piano, it is a safe prophecy that the sum-strength of the three white notes would greatly exceed that of the three black ones, however careful the playing; and the difference would be due to the position of the piano-keys and the leverage applied to the hammers.

(2) **Psychological.** There is a subtle kind of mental analogy that leads us to connect sharps with a rise and flats with a fall. If we play the chord of F major, and suddenly change it to F sharp major, we experience a feeling of lift and exhilaration which is quite absent when we settle to change it, not to F sharp, but to G flat major. Consequently all pianists and organists agree in attributing increasing 'brightness' to keys as the number of their sharps increases, and in feeling that keys acquire a subdued and less exhilarating

character in proportion to the number of their flats. But the feeling, whether of exhilaration or depression, is caused solely by the performer's mental processes, and is not inherent in the key.

A striking instance of this psychological bias was furnished recently by a great performer, who said that when he played Bach's C sharp major fugue (from the 48, Book I) from an edition in which it was printed in D flat, the whole composition changed its character. Every musician will understand exactly what was meant by this statement; but none the less it is clear that the only change possible under the conditions is a mental change in the performer, since the listener will not know which edition is being used.

It is a curious and little-noticed fact that violinists entirely disagree with pianists in the characteristics they attribute to keys. To them the brightest of all keys is G major, and they will readily play a passage to prove their contention. But if they are asked to play the same passage, still in G major, on a violin tuned up or down a semitone, they admit at once that the brightness is gone. So the conclusion is inevitable that to violinists the questions of pitch and key-signature are relatively unimportant, for a passage in the dullest key they could mention becomes 'bright' as soon as we tune the open strings to suit that key as they normally suit the key of G.

Finally, there can be no doubt that Association of Ideas must play a part in the matter. When it is once generally felt that any key has a certain predominant characteristic—that E major, for instance, is a cheerful key—not only will we be apt to attribute cheerfulness to music in E major, but composers will tend to write cheerful music in that key rather than in others. And so every new example will accentuate our impression, until the cumulative effect of many examples will lead us to associate the quality and the tonality as two things inseparable in our past experience.

It is unlikely that any dogmatic statement will be acceptable to those who have made up their minds as to the characters of keys. Yet it would seem possible to say, without presumption, that any character we impute to a key does not really belong to the sounds of a composition in that key, but is the result of what we ourselves feel, owing to the psychological effect of the key-signature. It is not a matter of pitch, for those with no sense of absolute pitch are by no means the least determined in the matter; and it is not due to the accidental fact that the arrangement of black and white piano-keys affects uniformity of sound, since people who do not play the piano feel key-character as strongly as those who do. The general verdict of a number of musicians, appealed to on the matter, agreed in attributing 'brightness' to F sharp major, and 'mellowness' to G flat major; and all agreed that, were two copies made of a composition, one in each key, the player would feel one or other of these qualities predominating, according to the copy which happened to confront his eyes. As the instrument would produce exactly the same sounds in either case it seems as certain that key-character is due, in the case of keyed instruments, to psychological causes (which means that it is really purely imaginary, imposed on a key by the player and not inherent to the key), as it is that, in the case of stringed instruments, it is due to the influence of the open strings and their partials.

PART VI. TRANSMISSION

CHAPTER XV

ON THE NATURE OF MOTION

THE word Motion, to those who have never had occasion to consider it carefully, implies the movement of an object involving the change of its position. When a ball has been thrown from one person to another, when a shot from a rifle has hit a target, when a man has left his house and arrived at the station, the most unsophisticated mind apprehends that there has been motion, since the ball, the bullet, and the man, which began by being in one position, end by being in another.

In addition, there will be a general recognition, even amongst elementary minds, that different forms of direct motion possess different characteristics. Every one will realize that the man may walk to the station at a uniform pace, or may vary his speed—walking slowly up hill and fast down, with an extra spurt at the end on seeing that his train is signalled; most of them know that the pace of the bullet decreases as it flies, and that if it hits a human being his chance of being hurt lessens as his distance from the rifle increases; some of them may have discovered that the ball, after reaching a certain height, curves downwards towards the earth because of the influence of gravitation.

All such knowledge, however, is concerned with an aspect of motion which involves the removal of something solid from one place to another. And even when people have become curious enough to propound questions on the subject, their riddles will usually take a form implying the same conception: such as the familiar problem as to whether a man is really moving when he walks along a ship's deck at exactly the same pace as that of the ship's motion, but in the opposite direction.

Now the study of Sound is really the study of three different things which can and ought to be kept in water-tight compartments. They are:

(1) The study of the cause or origin of air-vibrations;
(2) The study of the transmission of air-vibrations from the origin to the ear;
(3) The study of the mechanism of the ear which transmutes the stimulus received in the shape of air-vibrations into the sensation of Sound.

The motion involved in the first of these three sections is of the kind just discussed. That is to say, when a vibrating body vibrates we do actually have an instance of something solid changing rapidly from one position to another. But with this chapter we begin the study of transmission, section (2) in the above division, and the transmission of sound is a question of motion and of nothing else; but it is motion of a nature almost entirely different from that with which we have hitherto been dealing.

This new conception of motion, called Apparent Motion, is not in any way difficult to grasp, being merely a new way of looking at facts perfectly obvious to every one. But it is so imperative that a student, on beginning to consider Transmission, should have a clear and distinct idea of the change which occurs at this point in the meaning he gives the word, that the nature of Apparent Motion will now be analysed in detail.

Apparent Motion. The easiest way to get a preliminary idea of Apparent Motion is by means of an example. Imagine a long row of ladies of the same height drawn up in a straight line, each one touching her neighbour. They have been told, let us suppose, that Royalty is going to drive past them, and that each in turn must make her curtsy. If you then take up a position behind them, at such a distance that you can see the row as an unbroken line without seeing that it is composed of individuals, you will notice, as soon as the curtsying begins, a depression sweep along the line from one end to the other. Something, that is to say, has moved along the top of the line from left to right or right to left, whilst the only actual movement of any individual unit has been directly up and down. If we consider the movements as taking place in a plane at right angles to the earth, the direct motion—i. e. the various curtsies—has been north and south, whereas the apparent motion has been east and west; and we have an elementary example where the motion of particles is *at right angles* to the motion of the whole.

A second example, in which the motion is the same as that just described, is the ordinary sea-wave. A series of waves may be seen, all clearly proceeding in one direction, at a time when we know that the actual water does not change its position at all. Any one drop of water simply moves up and down almost [1] perpendicularly, whilst the wave-form moves onward over the surface; and so we have another instance where the direct motion is at right angles to the apparent.

A third illustration of the same thing in a less obvious form is furnished by a rope fixed at one end and jerked up and down by some one who holds the free end. The result is

[1] The student need not worry, at this point, about the word 'almost', which is inserted owing to the discovery of Weber's Law. This established the fact that the actual path of all particles on the surface of the water is oval; and that the oval approximates to a circle as the depth of the water increases. It is consequently assumed that when the depth of the water is very great the path of each surface-particle becomes circular. The student should, of course, be acquainted with Weber's Law, but for the purpose now in hand (i. e. the realization that the forward movement of a wave is, *qua* movement, quite distinct from the movement of the drops of water which constitute the wave) no harm is done by considering the motion of surface-particles to be just 'up and down', like the curtsying of the ladies in the previous illustration.

a snake-like curve in the rope, always travelling away from the holder and towards the fixed end. Yet a moment's thought discloses that any one particle of rope is travelling up and down in a line perpendicular, for all practical purposes, to the ground—i. e. at right angles to the apparent motion of the rope-wave.

One more example is necessary, for the purpose of introducing a new consideration. A field of wheat is standing, on a calm day, with all its wheat-stalks perpendicular to the ground (fig. 39). A gust of wind strikes the wheat on the extreme edge of the field and blows the stalks of fig. 39 into the shape of fig. 40. A moment later the stalks affected regain the perpendicular position by reason of their elasticity, as the passing gust is bending another set of stalks (fig. 41).

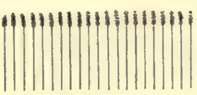


FIG. 39

FIG. 40

FIG. 41

In this example the apparent motion is that of a wave travelling over the whole surface of the corn. The direct motion is that of the corn. Two things should be noticed:

(1) The ears of corn move, not at right angles to the direction of the waves, but practically in the same plane. But half of their movement (i. e. when they are regaining the perpendicular) is in the *opposite* direction to the apparent motion—i. e. the motion of the wave.

(2) Any given number of ears are alternately compressed into a space less than that normally occupied (as in fig. 40) and extended over a space greater than normal (as in fig. 41).

Such apparent motion as has been described is, it will probably be admitted, not difficult to comprehend. But comprehension becomes less easy when we find the same process at work in two other ways which, though without thrusting themselves on our notice, are far more frequent.

The first of these two cases is when the medium is invisible. In water-waves, rope-waves, and corn-waves we can see with our eyes the apparent motion of the medium—the water, rope, or corn. But when the medium is air our understanding lacks the help of the eyes.

The standard experiment is as follows: Take a long tube—say a pipe of two-inch diameter—and place a lighted match at one end, close to the opening. If the hands are sharply clapped close to the opening at the other end the flame will go out exactly as if it had been blown.

Many people, on witnessing this experiment, are apt to think that the shock at the end of the pipe drives the column of air *through* the pipe—squeezing out so many inches at the other end just as ointment is squeezed out of a tube—and that these inches puff out the flame in the same way as a column of air directly blown from the mouth. But as a matter of fact the displacement of air in the tube is very small, and the extinguishing has been done by the shock which has travelled along the tube *through* the medium of the air enclosed; while the actual movement of any given particle of the air has, during a portion of the time, been in a direction directly opposite to that in which the shock has been travelling. The particles of air have been in alternate conditions of condensation and rarefaction, exactly as the ears of corn were in alternate conditions of compression and extension, and every particle of air has, at some moment between the instant of clapping and the instant of extinguishing, been travelling, at least once, *away* from the flame and *towards* the hands.

The second case is when the medium, though visible, does not perform any movement which the eye can detect.

The stock example is that of the transmission of the motion of a billiard-ball through a number of stationary balls. Place half a dozen billiard-balls in a row, touching each other, and all touching the cushion; then place a seventh ball a little distance from one end and in the same straight line—i. e. touching the same cushion. If the seventh ball is then hit on to the ball nearest to it—No. 1 of the string of six—the result will be that it will stop dead, whilst the ball at the other end breaks away and runs along by itself, leaving behind it a stationary row of six balls, all touching each other and all touching the cushion.

The experiment needs a certain amount of regulation in practice, since it is not difficult to hit the seventh ball so hard that the whole string of six balls is disturbed. But for our purpose we may ignore everything except the one fact that it is possible for the first ball, when hit properly, to stop dead and project its impetus, though five other balls all apparently perfectly still, on to a sixth ball which, finding no barrier in front of it, starts on a journey of its own.

To examine what actually occurs inside each ball during this transmission of energy would involve an inquiry into the constitution of matter. All that we need comprehend is that the particles composing the solid ivory balls do, without permanently changing their own positions with regard to any fixed point, manage to transmit, from one end to the other of five of them, a definite force which, on reaching the last ball, is translated into motion.

CHAPTER XVI

ON CURVES OF POSITION

EVERYBODY has, at some time or other, taken part in a discussion on a question involving the analysis or comparison of abstract things; and few can have failed to realize that the difficulties inherent in such discussions are due to the absence of standards of measurement. Musicians will at once recognize the difficulty as one that arises in all questions and arguments about Beauty. We may, perhaps, feel fairly safe in saying that we think one composition more beautiful than another; but if we describe it as 'twice as beautiful' the valuation becomes, in the absence of measuring-standards, a mere figure of speech.

Men of science, with whom the one essential condition of progress is exactness, discovered this difficulty at an early stage; and they set to work devising methods of measuring such things as motion, rates of increase, &c., graphically and exactly, and expressing the results of the measurements by means of curves. Their methods used to be considered so difficult to understand that only the most advanced mathematical students at any school in England were introduced to them. Nowadays, however, it is recognized that the elements of the matter are very simple, and small children are initiated, by means of 'graphs', into the first steps. As these steps are absolutely essential to the understanding of any facts whatever connected with the transmission of sound, the student will be wise to make himself entirely at home with the contents of this chapter, before attempting to deal with their application later on.

The essential apparatus of a Curve of Position is shown in fig. 42. On a piece of squared paper we draw two lines at right angles to one another, which are called Axes. We then settle that we will register the amount of one characteristic along the horizontal axis, the amount of another along the perpendicular axis. Both axes are already partitioned into equal sections, and we have to fix, before starting, *how much* of either characteristic one section will stand for; and the amount so chosen will be our *unit*.

To make the above process clear beyond any possibility of misunderstanding let us take the case of a butcher's bill. If beef is a shilling a pound we can register the relations between

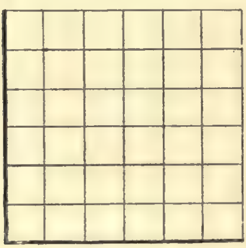


FIG. 42

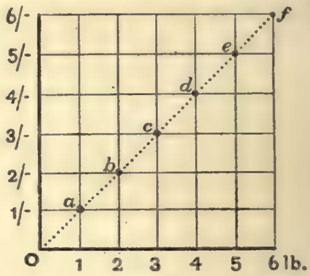


FIG. 43

weight and cost as follows. The horizontal Axis is to register weight, the unit (one section) being 1 lb.: the perpendicular Axis is to register price, the unit being one shilling. Starting at O (the *Origin*), we move one section east if we are dealing with one pound, and one section north because the one unit of weight will cost one unit of price; and we register our first point (*a*) where the perpendiculars from the axes intersect. Dealing in the same way with amounts of 2, 3, 4, 5, and 6 lb., we register the points *b*, *c*, *d*, *e*, and *f*; and by joining our points by the dotted line of fig. 43 we find that the 'curve of position' takes the form of a straight line.[1]

[1] Many students will recognize the familiar 'graph' of their schooldays. But the word 'graph' has been avoided, since the introduction of another technical term might be confusing.

One case of the use of this form of registering is familiar to every one—the temperature-chart. Here the horizontal axis represents time, the unit being generally four hours—so that the six sections of fig. 43 would equal one day; while the perpendicular axis represents temperature, each section usually representing the unit of one-fifth of a degree Fahrenheit. The patient's temperature is taken at fixed intervals, and marked on the chart; and when these marks are joined we get the Temperature-curve.

The non-mathematician may be puzzled at this point by the fact that the word 'curve' in both our examples is used in an apparently wrong sense. In fig. 43 our curve was obviously a straight line, while a temperature-curve will clearly be a zigzag composed of straight lines. He must, however, accept two facts which cannot be discussed here, and may rest assured that, though they may seem to him contrary to common sense, their universal acceptance as axioms by all mathematicians does not really betray any stupidity:

(*a*) All curves are really composed of an infinite number of straight lines, since all motion from a point of rest must be momentarily in one definite direction.

(*b*) A straight line is only a special form of curve.

The same methods of registration can be applied, with equal simplicity, to problems of elementary motion. If we wish to 'plot out' the curve of position for a man walking at the uniform rate of four miles an hour, we draw our axes as before, and then fix our units of time and distance. If we choose fifteen minutes and one mile we get the result in fig. 44; if we prefer one hour and two miles we get the result in fig. 45.

Two conclusions should be evidently true to any one who really understands what has been said up to this point:

(*a*) When motion is *uniform* the curve of position is invariably a straight line.

(*b*) The curve of position is never, under any circumstances, intended to illustrate the path actually travelled. The man in figs. 44 and 45 *may* have walked in a perfectly straight line, but he may equally well have been walking round and round the Albert Hall. This fact is of essential importance in understanding the Associated Wave.

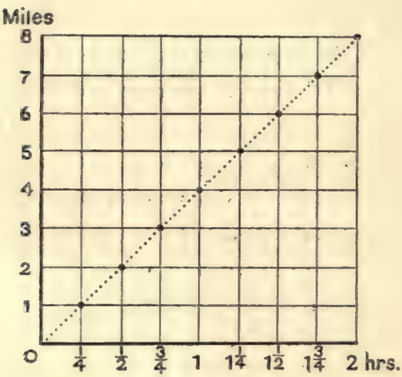


FIG. 44

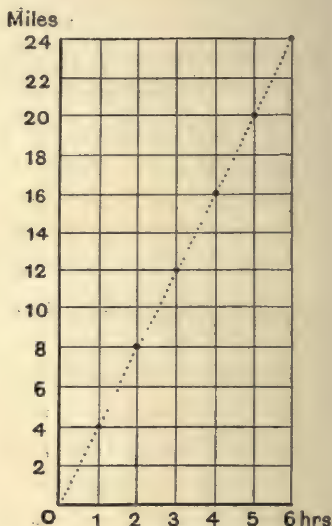


FIG. 45

Up to this point all our examples have illustrated increases that were uniform, and all our dotted lines have, in consequence, been straight. We will now take an instance where the result is otherwise.

An engine starts from rest at a station. Let us suppose that it increases its speed at a uniform rate, so that during the first minute it travels 100 yards, during the second minute 200 yards, during the third 300 yards, and so on, until its maximum speed is attained.

If we wish to plot out its curve of position we must draw our axes as before, and fix our units of time and distance.

Let us decide on units of one minute and one hundred yards, as in fig. 46. At the end of one minute the distance travelled is 100 yards, and the point *a* registers it.

At the end of the second minute, during which it has travelled 200 yards, it will be 300 yards from the station—registered at *b*.

The third minute will take it 600 yards from home, at *c*, the fourth minute 1,000 yards, at *d*.

It is clear that the four points marked in fig. 46 cannot be in the same straight line; but it may appear to some that from the origin to *a*, from *a* to *b*, *b* to *c*, and *c* to *d* must be straight lines, and consequently that the result of joining them is no true curve. But this is not so. If a straight line from O to *a* represented the engine's position, then its speed would be uniform, and we could see that it must have done 50 yards in half a minute. But the engine's speed was not uniform—it was the *increase* of speed that was postulated as uniform, and to represent this the dotted line of fig. 46 must be *continuously* bending, to show that at any moment the speed is slightly greater than at the moment before.

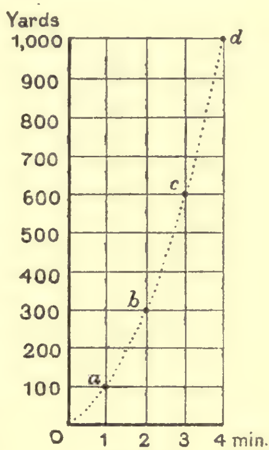


FIG. 46

It is so important that this point should be clearly understood that it will be well to analyse the movement of the engine, together with its curve of position, during the first minute—i. e. the portion of the curve from the origin to *a* in fig. 46. Let fig. 47 represent the bottom left-hand square of fig. 46 in an enlarged form: OA being one unit of distance (100 yards), OB one unit of time (one minute). Now, if the engine travels at a uniform speed of 100 yards per minute, with no acceleration, its curve of position is obviously represented by the diagonal OP. Should we want to find out how long it has taken to reach a certain point p, we drop a perpendicular pN (called the *ordinate* of p) to the time axis and measure ON. If it is the distance travelled, and not the time occupied, that we want to find out, then we draw a perpendicular (called the *abscissa*) to the distance axis and measure OM.

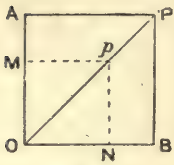


FIG. 47

It is evident from fig. 47 that so long as the engine covers one unit of distance in one unit of time, *at a uniform speed*, the ratio of the ordinate to the abscissa will always be the same; but that as soon as we introduce the idea of *acceleration* the ordinate increases in length quicker than the abscissa, and so the same ratio continuously increases in value.

Consequently, if we insert p in fifty-nine places between O and P, giving the position of the engine at the end of each second (the position of p at the end of the sixtieth second coinciding with P), we shall get, by joining the sixty p-points together, not the straight line OP but a curve. [It is true this curve would, on paper, consist of sixty diminutive straight lines, but we are not limited in the number of points we can take, and so we can make these straight lines as small as we like; and when they are infinitely small the result is a curve.]

The only curve with which students of Acoustics from a purely musical standpoint *must* be familiar is the one which shows, in the manner of those already described, the rate of increase and decrease of the speed of the bob of a pendulum during its swing. Suppose that we are watching a child swinging in the garden, and wish to put on paper something that will represent the motion of the child. It would be quite simple to draw a semicircle with its ends pointing upwards (fig. 48), and to say that O is the fixed end of the swing, S the swing when at rest, and the semicircle the path travelled by the swing when in motion. For, since the length of the swing-rope is constant the swing must travel some portion of the arc ASB. It would, however, scarcely be worth our while to make such a diagram, unless we were teaching a class of small children, for every one knows, without pictorial illustration, that the path of a swing is circular.

But we know also another fact, viz. that the swing is momentarily at rest at B, falls with ever-increasing speed to S, where its maximum speed occurs, and then rises with ever-diminishing speed to its opposite position of rest at A; and we may desire to put on paper something which will show the variation in the rate at which the child moves.

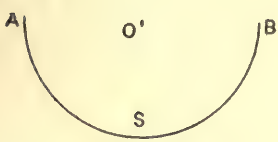


FIG. 48

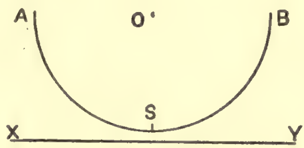


FIG. 49

The problem may be stated in another, and possibly simpler, way. Let fig. 49 represent a swing exactly similar to that in fig. 48, and suppose that the sun is shining immediately overhead. As the child swings from A to B a shadow will be thrown on the ground from X to Y. This shadow will begin by moving slowly from X, but at a greatly increasing pace, until the child reaches S, the point of maximum speed; and then the shadow will travel for the second half of its journey at a pace exactly the reverse of that of the first half. The motion of the shadow is known as simple harmonic motion; and the curve required is one which will represent the variation in speed.

This curve (fig. 50) is known as the curve of sines, and its shape is exactly what would be drawn by the shadow (if we can imagine the shadow being provided with a pencil) on a piece of paper drawn across its path at a uniform rate and at right angles to the direction of its motion.

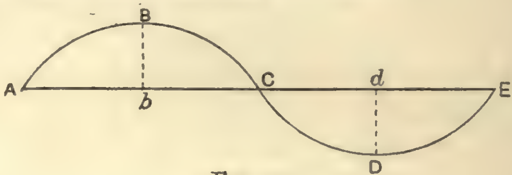


FIG. 50

In the above curve

AE is the *length*;

ABC is the *crest*, CDE the *trough*;

Bb is the height of the crest, Dd the depth of the trough, and they are equal;

AC is the length of the crest, CE the length of the trough, and they are equal;

Bb (or Dd) is the amplitude of the curve.

[Readers whose mathematics includes a knowledge of the meaning of the trigonometrical term *sine* may get a firmer grasp of the curve of simple harmonic motion from the following explanation.

If we suppose that in fig. 51 the point P is travelling at a uniform

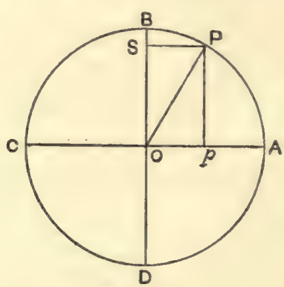


FIG. 51

speed from A to B, then its distance from the diameter AOC is always represented by the perpendicular Pp. Now the length of Pp compared to OP (which, being a radius, is constant) is a ratio depending on the angle POA, $\frac{\text{P}p}{\text{OP}}$ being, in fact, the sine of that angle (sometimes it is called the sine of the arc AP). Since P moves at a uniform speed we may consider the arc AP proportional to the time, and then P*p* (which is always equal to the distance of S from the centre if PS be drawn perpendicular to BO) will represent, by its increase and decrease, the variation of pace in the movement of S along OB when DOB corresponds to the line XY in fig. 49 and S in the shadow travelling along it.

This movement of S along DOB exactly corresponds with the motion of a single particle of air when the mass of air is excited by a vibrating body into alternate states of condensation and rarefaction; and those who are able to grasp the above description of a circular function will be able to feel that their grasp of the process is founded on mathematical truth.]

CHAPTER XVII

ON THE ASSOCIATED WAVE

WHEN a vibrating body is in motion it communicates its vibrations to the air, which, in its turn, communicates them to our ears; and we say that the vibrations have been 'transmitted' through the air from the vibrating body to the listener. We have now to examine what happens in the air itself whilst vibrations are in course of transmission.

It has already been stated that air has a power of expansion and contraction possessed by few other substances. If we pour half a pint of water into a pint-pot the pot is only half filled. But if we make a vacuum in the pot and let in half a pint of air, *the air fills the pot*; though in expanding itself to twice its normal volume it halves its density.[1] In the same way a pint of air can be compressed into a half-pint receptacle, in which case the density of the air will be doubled. And when the density is *diminished* we say the air is in a state of rarefaction, when it is *increased*, in a state of condensation.

It would be possible, by way of illustration, to construct an air-tight telescope in which the air was in a normal condition when the telescope was exactly half-way between its extreme and its shortest length. Then, by alternately opening and closing the telescope, we should throw the air into alternate states of rarefaction and condensation; and the amount of rarefaction when the telescope was fully extended would be equal to the amount of condensation when it was closed tight, since each state would be equally removed from the normal.

It is possible to give some idea of these alternate states of

[1] See 'density' on p. 13.

condensation and rarefaction by means of shading. If we take a sectional view of the air between a vibrating body and a listener we can imagine it as in fig. 52, where the dark lines signify compression and the lighter intervening spaces rarefaction at a particular moment.

It must be understood that the movement in the air illustrated in fig. 52 is not really circular but *spherical.* We could draw arcs from the tuning-fork to reach ears on the west, north, and south, as well as the one on the east; but the vibrations would also reach ears on this side of the paper and on the other side as well. The illustration shows in two dimensions a process which really involves three, but since

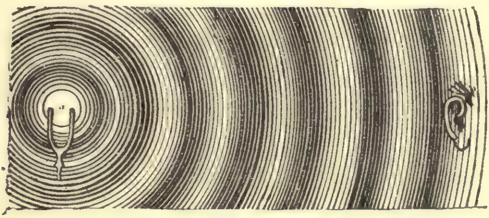


FIG. 52

exactly the same thing is happening in any straight line from the origin of a sound—along any radius, that is, from the vibrating body to the circumference of its sphere of vibrations—we can still further simplify the matter by reducing the two dimensions of fig. 52 to one, and consider merely what happens along any straight line between the origin of sound and the ear.

If the last paragraph is not clear the student may find help in the following analogy. Fig. 52, if completed, would show a series of circles. But for a true picture of air-vibrations—since they spread equally in all directions—we should imagine a series of footballs of different sizes, one outside the other. We want to find out what is happening inside between the centre and the outmost bladder. But what is happening between the centre and *any* of a hundred different points on the outside bladder is exactly the same: wherefore if we examine the straight line between the centre and any *one* point on the outside we shall get a true conception of the whole process, and can ignore the fact that it is a spherical process altogether.

More obviously still, when a man on a platform sings a note the air-vibrations start on their journey in all directions at once, i.e. spherically. But between the singer's mouth and the ears of the various listeners are a number of straight lines, and the air is doing exactly the same thing along all of them. So it is only necessary to examine one of these lines to conceive the whole system.

The object of the Associated Wave is to represent to the eye the process of rarefaction and condensation of the air along such a line.

A glance at fig. 52 will show that a point may be taken, half-way between the extremes of condensation and rarefaction, where the state of the air is normal. It then becomes (1) more and more rarefied until it reaches the maximum of rarefaction; then (2) becomes less and less rarefied until it again reaches normal; next (3) it becomes more and more condensed until it reaches its maximum of condensation; finally (4) it again proceeds to normal.

It is an established fact—and no musical student need enter into the calculations which lead up to it—that the rates of increase and decrease of rarefaction and condensation just described are exactly proportional to the rates of increase and decrease of pace in the shadow thrown by the swing along the line XY in fig. 49, p. 119.

Hence the wave-curve illustrative of the rate of increase and decrease of pace of a pendulum (fig. 50, p. 125) is also the curve which shows pictorially the process of rarefaction and condensation in the air caused by a pure musical sound; and this curve is called the **Associated Wave.**

Certain facts must be remembered about this curve and its proportions, but the most important of all is the fact that no motion whatever takes place in the air (or anywhere else) remotely resembling the *shape* of the curve. The air does not vibrate in curves at all, but in throbs, backwards and forwards along a straight line ⟷ ; and the sole object of the curve (as of all curves of position) is, not to draw a picture of what happens, but to give a graphic representation of those variations in its density, due to rarefaction and condensation, which cannot be exactly shown in any other way.

(*a*) In fig. 50 the length of the wave is AE, and this governs the pitch. If we find the length is 10 feet, then, taking the velocity of sound at 1,100 feet per second, the frequency of the note is $\frac{1100}{10}$, or 110 vibrations per second. Conversely, the note whose frequency is 260 will have a wave-length of $\frac{1100}{260} = \frac{55}{13}$ feet.

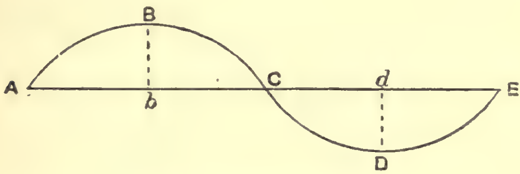


Fig. 50 (reproduced).

(*b*) The crest ABC corresponds to the condensation, the trough CDE to the rarefaction of the air. The height of crest and trough are measured by the amplitudes B*b* and D*d*, and these, in the case of pure musical sounds, will always be equal to each other; for since the air in any wave-length is a given quantity any condensation of it in the half AC involves a compensating rarefaction in the half CE.

(*c*) The amplitude of the wave corresponds to the intensity of the sound it represents. A sound increases in volume in exact proportion to the condensation of the air at its maximum point; and the sole duty of the line B*b* is to register the maximum density.

[Readers who like to reduce their knowledge to formulae may be interested in the following mathematical statement.

The vibration-number of a note is found when we divide unity by the period required for one complete vibration. If one vibration takes one-hundredth of a second the frequency of the note is 1 divided by $\frac{1}{100}$, i.e. 100. If we call the frequency f and the period p, we get the formula

$$f = \frac{1}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Again, since the velocity of sound is always equal to $\frac{\text{distance}}{\text{time}}$ we may say that the velocity of a wave is $\frac{\text{wave-length}}{\text{period}}$, for a period means simply the time which a vibrating body needs for one complete swing; i.e. the time which a complete vibration takes to pass through the air. Calling wave-length l and velocity v, we get another formula:

$$v = \frac{l}{p} = l \times \frac{1}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Substituting the f of (1) for the $\frac{1}{p}$ of (2), we get

$$v = lf \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

For example, the vibration whose period is $\frac{1}{100}$ second has a frequency of 100, and if we take the velocity of sound as 1,100 feet per second we know from (3) that $1100 = 100l$, and therefore the wave-length is 11 feet.]

The student must beware of making one very common mistake in his conception of the Associated Wave. The height of the curve at any point *does not represent the state of the air immediately* (i.e. *perpendicularly*) *under that point.*

Fig. 53 represents half the associated wave of a simple vibration. It is often carelessly assumed that the height pp^1 corresponds to the amount of condensation at the point p.

Now p is a position in the normal line, occupying the place proper to it when the air is at rest. If fig. 53 represented a *water*-wave the particle of water at p would travel upwards to p^1 and back again to p during the time that it formed part of the crest of the wave. Similarly, when the air is in motion p moves along AB to the point p^2, the same distance away from p as p^1. Thus, wherever we take p^1 on the crest, it represents the condensation at a point p^2, to be found by making $pp^2 = pp^1$; and the same is true of the trough, remembering that p^2 will be on the other side of p.

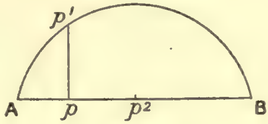


FIG. 53

The student will probably understand by now the statement that the Associated Wave expresses by transverse means vibrations in the air which are really longitudinal.

We have now to consider the character of the Associated Wave when the vibrations are not those of a simple musical sound.

The study of partial tones teaches us that a Clang is always composed of a prime with overtones, all of them, taken separately, simple tones.[1] Let us take a very simple Clang—one in which the prime is accompanied by its first overtone only. The frequency of the prime will be half that of its overtone, and consequently its wave-length will be twice as great. So that we know there must be, between our ears and the origin of sound, two sets of vibrations whose frequencies and wave-lengths are in the ratio 1 : 2. If AB (fig. 54) is the wave-length of the prime then the associated wave of that note will exactly cover its length; and if CD (fig. 55) is the same length, it will carry two associated waves of the first overtone.

The problem before us is this. What happens in the air

[1] See Fourier's Theorem, p. 74.

when these two sets of vibrations, of which figs. 54 and 55 are the Associated Waves, are being transmitted simultaneously? The answer is that they combine into one com-

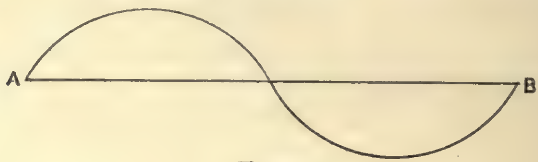


FIG. 54

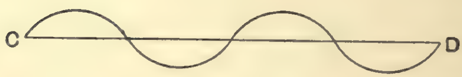


FIG. 55

plex vibration-system, whose associated wave is a combination of the two waves drawn above.

This combination-wave is found by drawing both simple waves to the same normal-line (or axis), and combining the ordinates at any point. If both ordinates are on the same side of the axis they are *added*; if they are on different sides the lesser is *subtracted* from the greater.

For example, fig. 56 shows one complete wave and the crest of a second wave:

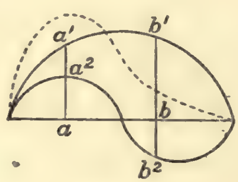


FIG. 56

At the point a the ordinate of the longer wave is aa^1, the ordinate of the shorter aa^2; both ordinates are on the same side of the axis, and so the amplitude of the combination-wave (the dotted line) will be the *sum* of them.

At the point b the two ordinates bb^1, bb^2, are on different sides of the axis, consequently the amplitude of the combination-wave will be the *difference* between them.

What is done in fig. 56 with two vibration-systems can obviously be done with a thousand; for any two simple waves can be combined into one resultant, and any two resultants combined in the same way.

A certain number of side-issues arise, in considering the question of Transmission, of which the most important are the following:

(*a*) The velocity of sound in air, though affected (see p. 16) by a rise or fall of temperature, is not affected by atmospheric pressure. If the temperature is constant the velocity is independent of the barometer.

(*b*) Variations in pitch and intensity, though they affect the distance at which a sound is audible, do not affect the velocity at which it travels.

(*c*) The pitch of sound is affected by any violent movement of the vibrating body to or from the listener. No musician can have failed to notice that a motor-horn drops in pitch as it passes him. The reason (called Döppler's Principle) is that as the car travels towards us each vibration, instead of reaching the ear at its proper distance from the previous one, is forced a little nearer to it. Thus the wave-lengths are shortened and the pitch raised; the reverse taking place the instant the car has passed us. Neither note is the true pitch of the horn, which lies half-way between the two notes.

(*d*) The effect of wind on sound-waves is curious. It does not affect their pitch, but considerably alters their velocity and range. If the surface of the earth were perfectly smooth and without interruption a favourable wind would carry a sound both farther and quicker than an adverse one, which latter tends to disperse and destroy vibrations. As things are, however, the wind travels at a slower pace near the earth than higher up, owing to obstacles and friction. Consequently sound is *refracted*, since in a favourable wind it will travel faster at a little height than near the ground, and the plane of its movement will be tilted towards the earth. In an adverse wind the lower vibrations, meeting with less resistance, travel faster than the higher ones and the plane is tilted up, with the result that the whole vibration-system is directed into space.

On a sultry day refraction occurs in the same manner, for the air near the earth becomes hotter than that above; it expands, diminishes in density, and causes sound to travel faster near the ground. At night the earth cools, and the lower air becomes in turn more dense than the upper, so that a travelling vibration-system is directed towards the ground instead of, as in the heat of the day, towards space.

CHAPTER XVIII

COMBINATION TONES

WHEN we listen to a single Clang we know that we are listening to a group of sounds varying in pitch. And even if the prime tone is the only one recognizable by the unassisted ear, we can prove the presence of other sounds by means of resonators.

When we listen to two sounds of different pitch another phenomenon presents itself. We can hear—or prove the presence of—sounds which are not to be heard when either note is listened to alone.

Such sounds are called **Combination** or **Resultant** Tones (occasionally known as *Tartini's Harmonics*), and the notes which cause them are called the **Generators.**[1]

There are two kinds of Combination Tones, called **Differential** and **Summational** Tones.

Differential Tones. The vibration-number of a Differential is found by the simple process of subtracting the frequency of the lower generator from that of the higher. Thus if two notes have frequencies of 200 and 150 they will, when sounding simultaneously, produce a differential whose frequency is 50.

It is usual, and simpler, to find differentials from a vibration-fraction rather than from the actual frequencies. In the above case, for instance, the vibration-fraction of the two generators must be $\frac{200}{150} = \frac{4}{3}$, and the differential will be $4 - 3 = 1$. Its pitch,

[1] Students who desire a physical explanation of the occurrence of Combination Tones are referred to any standard work on Acoustics. Only the facts which seem important from a musician's standpoint are dealt with here.

accordingly, will be that of the first partial in the series of which the generators are the third and fourth partials.

Fig. 57 shows the differentials of the concords within the octave. The upper stave gives the generators, with their vibration-fraction overhead; the lower stave shows the differ-

FIG. 57

entials, the numbers over them (being numerator of the fraction *minus* denominator) showing the place of the differentials in a series of partials which would include all three notes.

The Harmonic Chord is of the greatest mental assistance in finding the pitch of a differential, and whenever the numerator *minus* denominator happens to equal *one* the pitch of the differential coincides with that of the prime tone; but the student must beware of thinking of the two as being always the same. Should we, for instance, take an interval greater than an octave, the differential falls *between* the generators. The interval in fig. 58 has for its vibration-fraction $\frac{5}{2}$, and will generate a differential $5-2=3$. The pitch of this will be the G between the two generators, and this accounts for the fact that these two generators will always suggest to us the chord of C major (and not A minor), since, without knowing it, we are also listening to the note G.

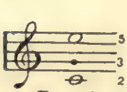


FIG. 58

A similar case has occurred within the writer's experience. The double-hooter at a neighbouring factory sounds a strong major 6th—say the G and E of fig. 58. When listeners are asked what key the interval suggests the invariable reply is 'C major', *never* 'E minor'. The reason is that these two notes, whose vibration-fraction is $\frac{5}{3}$, generate the differential $5 - 3 = 2$, which is middle C. But only *very* occasionally does a listener admit that he can hear the differential, though it is quite plain to any ear accustomed to listening for such things.

It is difficult to hear differentials when experimenting on the piano, partly because of equal temperament, partly because the tone of the piano is evanescent. But if a major 6th—such as middle C and the A above it—is struck loudly five or six times the F below can be heard faintly. And if, during the striking, the F-key has been depressed, the free strings will slightly reinforce the differential.

On the harmonium the sustained tone makes experiments more successful; and the concertina, with the double advantage of sustained tone and Mean-tone Temperament, is better still. But as only one really successful experiment is necessary to convince a sceptic it is probably most convenient for the ordinary musical student to use the violin.

If a minor 6th, G and B, as in fig. 59, is played loudly on the two top strings the differential D is plainly heard. Most violinists think this sound is due to the vibration of the open D string; but this is not the case, although in time the open string does reinforce the differential. But if the D string is damped by the finger at an early stage the differential is not affected; and if the notes are changed to A flat and C the differential rises a semitone at once without apparently losing any of its intensity.

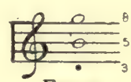
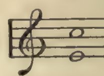
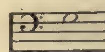


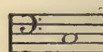
FIG. 59

Helmholtz invented an instrument called the Double Syren for the purpose of tracing differentials, and though a description of it would be too elaborate to undertake here every student who has access to a physical laboratory should make acquaintance with it. A few minutes' experimenting is sufficient to convince any one that differentials are not occasional accidents, but the invariable result of simultaneous sounds.

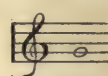
Differentials generated by two independent tones, such as we have been dealing with, are called *Differentials of the first order*. There are also *differentials of the second order*, generated between either independent note and the first order differential; and again *differentials of the third order*, generated between those of the second order and their predecessors, and so on. Thus if we take a minor 6th  (whose vibration-fraction is $\frac{8}{5}$) we get the following results:

Differential of the first order: $8-5=3$ 

Differentials of the second order: $8-3=5$ (already sounding)

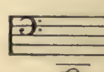
$5-3=2$ 

Differentials of the third order: $8-5=3$ (already sounding)

$8-2=6$ 

$5-5=0$

$5-2=3$ (already sounding)

$3-2=1$ 

Three considerations are worth noticing:

(1) Differentials are produced *amongst themselves*, and not between one of themselves and any overtone.

(2) Although any differentials but those of the first order are exceedingly difficult to hear, and become of less and less actual importance, yet their presence can be proved, and they are not mere theoretical inventions.

(3) Overtones do generate differentials of their own, but they can be ignored for practical purposes.

Summation Tones. These are sounds whose pitch corresponds to the sum of the vibration-numbers of two generators. Thus two notes whose frequencies are 200 and 150 will generate a summation tone whose frequency is 350.

These tones are extremely difficult to hear, and their importance is generally considered to be small; and if a student is aware of their existence and nature he will have no cause to inquire further in an elementary study of Acoustics.

Reference has already been made (p. 102) to the question of the harsh effects caused by clashing overtones in the playing of thirds on a tempered instrument. At this point we may draw attention to the difference, with regard to partials and differential tones, between the major and minor common chord.

When we hear the notes C, E flat, G sounded we also hear, if each note is a clang, the harmonic chord of each. Under certain circumstances a keen ear will find this chord very unpleasant, because of (as fig. 60 will show) (1) the 'curdling' of the overtones, and (2) the curdle between the prime E flat and the E natural which is such a prominent overtone to C. Composers did not take long to find out that a major chord was far more harmonious and smooth, since the only objectionable partials to E natural (the 3rd, b′ and the 5th g″ sharp) were not strong enough to be really offensive. Consequently they formed the habit of ending minor compositions with a major chord, and the major 3rd acquired the nickname of *Tierce de Picardie.*

Fig. 60

It is often forgotten that in the days when this custom grew up equal temperament was in its infancy. Composers wrote chiefly for the organ (tuned to Mean-tone Temperament) or for voices or strings which would sing and play true thirds.

In these conditions the change from an E flat to an E natural *which was in tune with the E already sounding as a partial of C* was a really important improvement. But nowadays, when even our organs are tuned with false thirds, it is doubtful whether the major chord could be considered in any way more harmonious than the minor, if the partials alone decided the matter.

When, however, we look at the differentials of the two chords it is easy to see that another element of great inharmoniousness is introduced. In the major chord (see fig. 61) all the three differentials of the first order have the pitch of

FIG. 61

C, and any one who will trouble to work out the differentials of the second order will find that they introduce no new note. The minor chord, on the other hand, gives us three differentials of the first order of which one is A flat—introducing quite a new discordant element; and the second order differentials introduce another A flat and a new note B flat as well.

We are therefore justified in feeling that, even in equal temperament, the smoothness of the minor chord is adversely affected by the differentials it produces, and so is less than that of the major chord which is not interfered with in any way from this cause.

Here to end

CHAPTER XIX

PHASE AND INTERFERENCE

IF we represent a certain distance of calm unruffled water—say 20 feet—by the straight line AE of fig. 62, and imagine that its surface is suddenly agitated by a system of waves 20 feet in length advancing from the direction of A, then the water will take some such form as the curve ABCDE.

But in drawing the above curve we have pictured the 20-feet wave at the moment it exactly filled the chosen 20-feet space; and it is obvious we might have chosen any other moment.

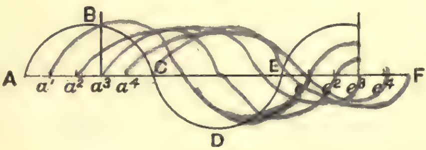


FIG. 62

If, for instance, we started drawing it at the point a^1, or a^2, the curve would be identical in shape with ABCDE, but would be shifted a little to the right and would end at e^1 or e^2.

The only difference between the various waves so drawn being their starting-place, they are said to be exactly similar waves in a different *phase*.

In the same way, the vibrations in the air between the origin of a simple sound and the ear are exactly similar, but if we could isolate a number of them for examination we might find we had one specimen just beginning its condensation, another just beginning its rarefaction. Provided, however, we had secured a full vibration-length in each case, every specimen would be a replica of all the others except for its phase.

That is, if we revert to the associated wave instead of the vibration itself, we might find one wave was the curve ABCDE, another the wave between the perpendiculars from a^3 and e^3.

Let us now examine the case of two simultaneous sounds which are in every way equal. They can be represented by two associated waves also exactly similar. If these sounds begin at the same instant then the resultant sound, as was pointed out in the previous chapter, will be represented by a wave of exactly twice the amplitude: that is to say, by the first law of Intensity (p. 46) it will be four times as loud.

Should one sound, however, start just that fraction of a second later than the other which would cause it to begin its condensation at the exact instant the earlier sound was

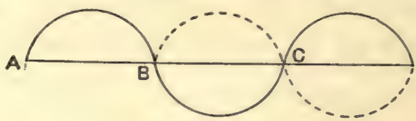


FIG. 63

beginning its rarefaction the result, expressed in associated waves, would be that of fig. 63.

In the above case the result, unexpected as it may seem, would be complete silence, and the phases are said to be in exact opposition.

It is, naturally, far more likely that the two phases will neither coincide nor be in exact opposition, so that the result of the two sounds will be something *between* silence and a volume of sound four times as great as either sound alone.

From fig. 63 it is clear that the dotted curve (representing the sound which starts late) may begin anywhere between A and C. If it starts at A the resultant has four times the intensity of the individual sounds; as it moves towards B the volume of the resultant decreases to zero; as it passes from B to C the volume of the resultant increases to four again. Thus there are only two exact instants at which the second sound can start in order that the resultant volume may be, what we should have expected it *always* would be, double the individual sound.

Students who meet for the first time with Interference (which is the technical name given to such phenomena as the above, when several vibration-systems occur simultaneously in a medium) are apt to regard it as purely theoretical. But it is open to any one to make acquaintance with its practical truth by experiments with a tuning-fork. When a fork vibrates the prongs separate and come together again. Consequently two systems of vibration are communicated to the air at the moment the prongs have come close together (i. e. the moment of striking):

(1) Rarefaction on the outside of each prong;

(2) Condensation at the back and front of the prongs, due to the compression of air *between* the prongs.

Fig. 64, in which the black oblongs represent the two tops of the prongs, will illustrate this:

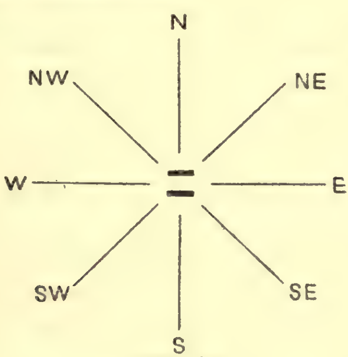


FIG. 64

As the tops of the prongs in fig. 64 come together a rarefaction is caused north and south; and at the same moment a condensation is thrown out east and west. And when the prongs fly away from each other the reverse process takes place.

Now these two simultaneous processes, caused by the same prongs, *must* start with phases in exact opposition. Consequently as a rarefaction spreads out from N westwards it will encounter an equal condensation travelling from W northwards; and along the line NW these two systems must cancel, as along the lines NE, SE, SW, also.

This is exactly what does occur, as may be proved by any one who will strike a tuning-fork, hold it vertically to his ear, and then revolve it. When a flat side is opposite the ear we have the full volume of sound, and when an angle presents itself we have either silence, or something sufficiently near silence to convince the listener of the reality of Interference.

When we think of a big chord sounded by a large orchestra, organ, and chorus, and remember that every individual sound made by every individual player and singer is producing different partials owing to difference in quality, we can form some notion of the bewildering complexity of the processes in the air which combine these myriads of vibrations into one resultant and bring this instantaneously to our ears for analysis. But *Fourier's Theorem* establishes the fact that such a resultant *can* be analysed, and the complementary *Ohm's Law* lays down that the ear has the required analytical power.

Helmholtz came to the conclusion that though difference of phase, by continually altering the amplitude of the resultant vibrations, does continuously affect the intensity of sound, yet it never affects the quality. From this he drew the inference that the vibrations of overtones do not cancel each other; that is to say, since any two sets of compound vibrations do not, on meeting, resolve themselves into groups of simple vibration, we can say that the simple vibrations of individual partials do not eliminate each other should they happen to be in opposite phases. It is probably safe to say, however, that this branch of Acoustics awaits further research before any conclusions can be considered as final; and what has been said on the subject of Interference, though little, embraces practically all that could be considered of interest to the musical student.

CHAPTER XX

CONSONANCE, DISSONANCE, AND BEATS

WE have now to examine what happens when two pure musical sounds, differing slightly in pitch, begin at precisely the same moment.

Consider what happens when two people walk together at the same pace, but with strides of different length. If a man and a child start walking simultaneously by putting their left feet to the ground at the same instant, and the child takes three double-steps (i. e. six strides) to the man's two, then as the man begins his third double-step the child will be beginning its fourth, at the same instant and with the same foot—the left. Their tracks in snow, so long as their pace and stride are uniform, will look like fig. 65 repeated *ad libitum*:

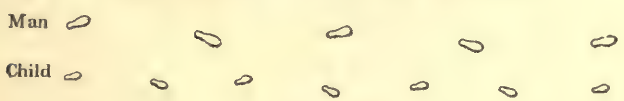


FIG. 65

The reader should notice the important fact that at the half-way point the diagram looks as if the walkers had fallen into step; but it is really the point where their phases are in exact opposition, the man putting down his left foot as the child steps on to the right.

Let us now examine wave-curves, taking the slightly more complicated case of two waves whose lengths are in the proportion of 6 : 5.

If we draw along a line AB a series of six uniform waves, and then along the same line a series of five, it will be clear (fig. 66):

(1) That the two wave systems are in opposite phases at the half-distance;

(2) That they cannot get into the same phase *between* A and B, but only at the points A and B.

FIG. 66

As soon as we consider the curves of fig. 66 to be the waves of two simple sounds we can see that between the coincidence of phase at A and the coincidence at B the resultant will start with an amplitude almost double that of either individual curve (in fig. 66 the amplitudes are equal), and will reach a point half-way where the amplitudes momentarily cancel, and finally will reach B when the amplitude is again almost double.

This means that when two sounds whose vibration-numbers bear the ratio 6 : 5 are simultaneously sounded with equal force the intensity of the resultant will not be a constant and steady quantity, but will throb between a quantity almost four times as great as that of either single sound and a quantity so small as to be practically silence.

Each such throb is called a *Beat.*

The number of beats caused by any two simultaneous sounds is found by subtracting their frequencies. If we take two imaginary sounds whose frequencies are 5 and 6, then fig. 66 shows that we should get one beat per second. If the frequencies were 10 and 12 there would be two beats per second, for obviously the curves of fig. 66 would repeat themselves, and two sets would represent the process of each second. Similarly two notes with frequencies 97 and 100 will produce three beats per second, and so on.

Many students find the above process a little difficult to grasp, and they may be assisted by the following illustration. If two garden swings are started together at different rates so that the longer completes 97 full vibrations while the shorter completes 100, it is clear that the two swings will start their 98th and 101st vibration at the same instant, and in the same direction as that from which they originally started.

Meanwhile the child in the quicker swing has caught the child in the slower three times; and on each occasion both children were moving *in the same direction*. And in the case of sound-waves these three moments (which would obviously be seven if the ratio of the swings was 93 : 100) will cause the throbs which we call beats.

On any instrument with sustained sounds, such as the organ or harmonium, it is very easy to hear the beats of small intervals, especially in the lower regions of the keyboard. When the lowest C of a harmonium is held down together with the semitone above it we do not hear a result of steady intensity, but rather a swirling and throbbing sound which will convince any one of the truth already enunciated: that the intensity varies between zero and a quantity four times as great as that of either individual sound. And though the throbbing becomes less noticeable as we widen the interval or raise the pitch, the process at work in the air is of an exactly similar nature.

Beats are sometimes intentionally used for definitely musical purposes; as in the *Voix Celestes* stop on the organ, in which each note is produced by two pipes *slightly* out of tune with each other. Tuners of instruments also make considerable use of beats, especially when gauging the pitch of very low notes.

In the case of pure musical sounds beats may arise

(1) between the tones themselves;

(2) between their combination-tones.

In the case of clangs beats may arise

(1) between the primes;

(2) between one prime and the overtones of the other;

(3) between the overtones of one and the overtones of the other;

(4) between combination-tones.

Beats may also arise between the overtones of a single clang, and it is usual to attribute harshness of quality to this cause. Good 'voicing' of an organ-pipe aims at eliminating such beats, and when it fails we get a stop with a certain blatancy. Every organist has come across a 'clarinet' which is only bearable when used together with a flute, i. e. with something which will affect the relative intensity of the overtones and so modify or obscure the beats which cause the harshness.

The effect of beats must be added to the causes already given (p. 135) in justification of the *Tierce de Picardie.*

All Dissonance is attributed to Beats.

When two pure musical sounds are in unison they produce no beats, but if we sharpen or flatten one of them beats immediately arise. So long as the number of these is small in comparison to the vibration-numbers of the notes the effect is not unpleasant—no one, for instance, objects to the Voix Celestes on the ground that it is harsh or dissonant; but as the interval between the notes is increased the unpleasantness asserts itself more positively up to a point, and then subsides until all trace of dissonance entirely disappears. The interval at which this disappearance takes place is called the *Beating-distance*, and in the region of middle C musicians are unanimous in fixing it at a minor 3rd. That is to say, if two instruments producing pure tones, and each tuned to middle C, begin vibrating together, no beats result. As soon as we sharpen one of the notes beats occur, but are not unpleasant so long as they are few in number. Unpleasantness, however, soon becomes definite, and the maximum of dissonance occurs at about the interval of a semitone; but when we reach the minor 3rd all dissonance has vanished.

At a lower pitch the beating-distance is greater than a minor 3rd, at a higher pitch less; and this explains the fact, which is common knowledge to musicians, that small intervals are less dissonant in the upper ranges than in the lower, and that a chord for trombones must not be 'spaced' as it might be for flutes.

In the case of two compound sounds, or clangs, the beating-distance cannot be placed at a minor 3rd in the region of middle C, because of the beats which may arise other than those of the prime tones.

Beats arising between the overtones of a single clang are of essential importance in determining quality; and amongst these the rule holds good that their beating-distance is a minor 3rd in the region of middle C. Sounds from stopped pipes are, as every organist knows, of singular smoothness, and when we consider that all the even-numbered partials are absent from them we see that the opportunities for beats are enormously restricted.

In the same way the string of a violin or 'cello will, if the bow is applied to the *centre* of the string, produce a tone so lacking in richness as to be almost dull. The reason is that when the centre point of the string is agitated it cannot be a node, and consequently all the even-numbered partials, which require a node at that point, are absent. Similarly, it is a matter of great importance whereabouts the hammer hits a piano-string, and many experiments have been made in quest of the point that will destroy only undesirable overtones.

On the other hand, all brass instruments owe their brilliance and penetration to the presence of upper partials in great number and strength; and these upper partials, being close to each other in pitch, produce amongst themselves beats in abundance.

It has been established for us by physicists that one of the conditions of consonance between two notes is that the vibration-fraction of the interval between them shall involve no odd number greater than five.

If we desire, then, to find the vibration-fractions of all the concords in an octave we have to find all the possible fractions between 1 and 2 which require no odd number greater than 5. These are:

$\frac{1}{1}$, $\frac{2}{1}$ (any others with denominator 1 will exceed 2).

$\frac{3}{2}$ ($\frac{2}{2}$ and $\frac{4}{2}$ merely reduplicate $\frac{1}{1}$ and $\frac{2}{1}$).

$\frac{4}{3}$, $\frac{5}{3}$.

$\frac{5}{4}$.

$\frac{6}{5}$, $\frac{8}{5}$.

From this table we can make a list of eight concords within the octave as follows:

1. Unison, $\frac{1}{1}$.
2. Minor 3rd, $\frac{6}{5}$.
3. Major 3rd, $\frac{5}{4}$.
4. Fourth, $\frac{4}{3}$.
5. Fifth, $\frac{3}{2}$.
6. Minor 6th, $\frac{8}{5}$.
7. Major 6th, $\frac{5}{3}$.
8. Octave, $\frac{2}{1}$.

The above intervals, perfectly concordant where pure sounds are concerned, acquire dissonance in varying degree when the sounds forming the intervals are clangs.

The unison and octave are free from dissonance because the harmonic chords are the same. This is obviously true of the unison, and in the case of the octave it is clear (fig. 67) that the partials of the higher note introduce no sound that would not find a place amongst the partials of the lower.

Fig. 67

Fig. 68

The interval which comes next in smoothness is the Fifth, though in this (fig. 68) there are obviously several dissonant partials present.

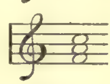
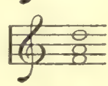
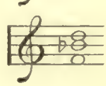
It is an interesting fact, though not of great importance to musicians (since in such matters they rely on instinct and experience), that the various consonant intervals were arranged by Helmholtz in order of comparative harmoniousness as follows:

1. Octave and unison.
2. Fifth.
3. Fourth, major 3rd, major 6th.
4. Minor 3rd.
5. Minor 6th.

Chords. If there are three notes in a chord, x, y, z, the smoothness of the chord will depend on three relationships:

x to y: lowest to middle note.
y to z: middle to top note.
x to z: lowest to top note.

When the vibration-fractions of these three intervals are all of them amongst the eight concordant intervals given on p.146, the resultant chord is a concord. Of these there are, within the octave, six and no more:

Lower interval	Higher interval	Interval of outside notes	Chord
minor 3rd $\frac{6}{5}$	major 3rd $\frac{5}{4}$	5th $\frac{3}{2}$	
minor 3rd $\frac{6}{5}$	4th $\frac{4}{3}$	minor 6th $\frac{8}{5}$	
major 3rd $\frac{5}{4}$	minor 3rd $\frac{6}{5}$	5th $\frac{3}{2}$	
major 3rd $\frac{5}{4}$	4th $\frac{4}{3}$	major 6th $\frac{5}{3}$	
4th $\frac{4}{3}$	minor 3rd $\frac{6}{5}$	minor 6th $\frac{8}{5}$	
4th $\frac{4}{3}$	major 3rd $\frac{5}{4}$	major 6th $\frac{5}{3}$	

CHAPTER XXI

THE CONSTRUCTION OF THE HUMAN EAR

It is only proposed to give the barest outline of the construction of the Ear, in order that the student may form a general idea of what happens to air-vibrations when they finally reach the listener.

The ear has three distinct sections, External, Middle, and Internal, and these will be dealt with in order.

External Ear. This consists of the Lobe, which collects vibrations, and a tube (about 1¼ inches long) down which they are directed. At the end of this tube the vibrations strike against the drum of the ear, or Tympanum, which closes the passage; and the Tympanum then passes the vibration system on to the middle ear.

Middle Ear. This is an air-chamber whose walls are almost entirely of bone. There is a passage to the throat (called the Eustachian Tube) and two small membrane-covered holes, one round, the other oval, called *Fenestra rotunda* and *Fenestra ovalis*. The upper one of these, the Fenestra ovalis, is joined to the Tympanum by a series of three small bones—the *Malleus* (attached to the Tympanum), which is sometimes called the Hammer bone, the *Stapes*, or Stirrup bone (attached to the membrane covering the Fenestra ovalis), and the *Incus*, or Anvil, which lies between the other two and is attached to both of them.

Every vibration of the Tympanum is faithfully conveyed by this series of bones to the membrane covering the Fenestra ovalis, and is in this way communicated to the Internal Ear.

Internal Ear. The construction of the Internal Ear is exceedingly intricate, and it is still a matter of speculation as to what are the specific functions of its individual parts. Roughly, we may say that it contains a membranous bag filled with a fluid called *Endolymph*, and that this bag floats in a fluid called *Perilymph*, which is kept out of the middle ear by the membranes covering the round and oval apertures.

The vibrations are communicated from the membrane of the Fenestra ovalis to the perilymph, and through the walls of the floating bag to the endolymph; and the end of the auditory nerve has its tentacles embedded in the latter. One membrane through which the vibrations must pass (called the *Basilar Membrane*) is composed of a set of fibres (the *Fibres of Corti*) of increasing lengths laid alongside one another rather like the strips of glass or metal in a child's Celesta. There are over 3,000 of these fibres, giving about 400 to the octave within the limits of recognizable pitch, and Helmholtz suggests that each is tuned to a note of a certain pitch and vibrates in sympathy with that pitch when the vibrations of such a note reach the inner ear.

Reference has already been made to Fourier's Theorem, which establishes the fact that any periodic vibration, however complex, can be analysed into a number of simple vibrations of definite relationship; and Ohm's Law establishes the fact that the ear can and does so analyse the complex vibrations presented to it.

If we were to mix together a large number of colours on a palette no human eye could, from a glance at the resultant, do more than guess at the presence of a few individual shades. If we mix together numerous kinds of food or drink, the most experienced taster could only, after many smackings of the lips, suggest a few of the constituents. But if we hear a big chord sounded by orchestra and chorus the average listener can immediately detect the presence of voices, organ, strings, brass, drums, and what not, and the trained ear of the expert will instantaneously and certainly declare so many facts about what he has heard in a fraction of a second that we may justifiably claim the ear to be, in accuracy of result not less than in rapidity of working, the most delicate and efficient organ of the human body.

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